Horizon Instability of the Extremal BTZ Black Hole

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Horizon instability of extremal black holes

Aretakis discovered a horizon instability of extremal black holes

Transverse derivatives of perturbations grow *at late times on the horizon*

$$\partial_x^n \underbrace{\psi}_{\text{Scalar field}} \sim V^{-P+n} f(Vx)$$

$$\text{as } \underbrace{V}_{\text{Ingoing time}} \to \underbrace{\infty}_{\text{Late times}} \quad \text{and} \quad \underbrace{x}_{\text{Radial coordinate}} \to \underbrace{0}_{\text{Horizon}}$$

On Horizon decay rate

$$V^{-P}$$

Off Horizon decay rate

$$V^{-2P}$$

2 Non-overlapping techniques

Aretakis – Conserved quantity on extremal horizons

Also Lucietti, Murata, Reall, Tanahashi

Initial data must extend to the horizon

'Discrete' case only (eg. massless, axisymmetric scalar in Kerr)

Gralla & Zimmerman – Mode sum approach using near-horizon geometry

Initial data is supported entirely outside the horizon

'Non-discrete' case only (eg. non-axisymmetric scalar in Kerr)

We hope to get more insight by studying a system with full analytic control - BTZ

BTZ rotating black hole

Banados-Teitelboim-Zanelli rotating black hole in 2+1 dimensions, asymptotically Anti-de-Sitter

3D vacuum solutions to EFE with cosmological constant – constant curvature $R_{\mu\nu} = 2\Lambda g_{\mu\nu}$

Negative cosmological constant => locally isometric to Anti-de-Sitter space

Periodic identification of a coordinate in AdS_3 gives the BTZ black hole

Why BTZ?

- Help understanding holographic signature of instability
- Only system where we have a closed analytic form of the retarded Green function
- Help fill gaps in understanding the instability

Coordinate patches of AdS_3

Global patch

Poincare patch

'Extremal' patch

Scalar field perturbation AdS_3 Green function

$$(\Box - \mu^2) G_{ret}^{\mathrm{AdS}_3} = \delta$$

$\Sigma \propto$ geodesic distance between 2 points in AdS_3

Cuts off advanced and spacelike separated points

Dirichlet boundary conditions – faster of two possible decays at boundary

$$G_{ret}^{\mathrm{AdS}_3} = \frac{\Theta(t-t')\Theta(\Sigma)}{\pi\sqrt{|\Sigma(\Sigma-2)|}} \begin{cases} -\cos\left(2\nu \arccos\left(1-\Sigma\right)\right) & \text{if} \quad \Sigma < 2 \\ \sin\left(2\nu\pi\right) e^{-2\nu \mathrm{arccosh}(\Sigma-1)} & \text{if} \quad \Sigma > 2 \end{cases}$$

Hadamard form

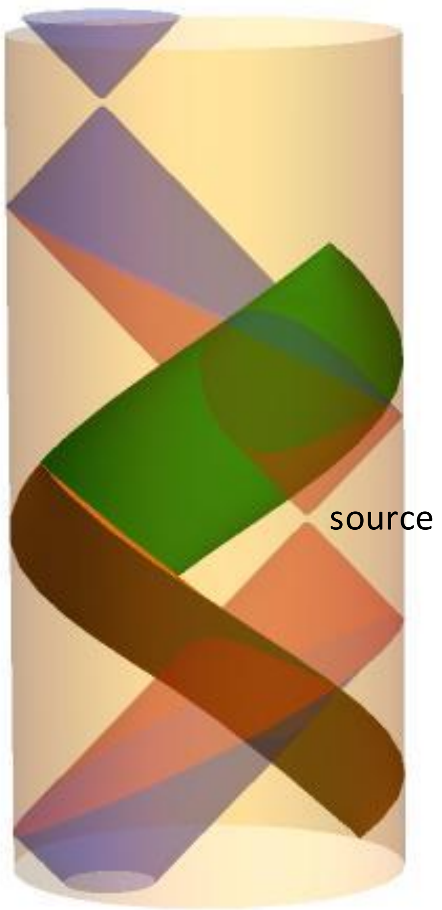
$$\nu = \frac{1}{2}\sqrt{1+\mu^2}$$

$2\nu \in \mathbb{N} \implies$ Discrete

$2\nu \notin \mathbb{N} \implies$ Non-Discrete

If $2\nu \in \mathbb{N}$ Green function has no support for $\Sigma > 2$

Boundary Bouncing



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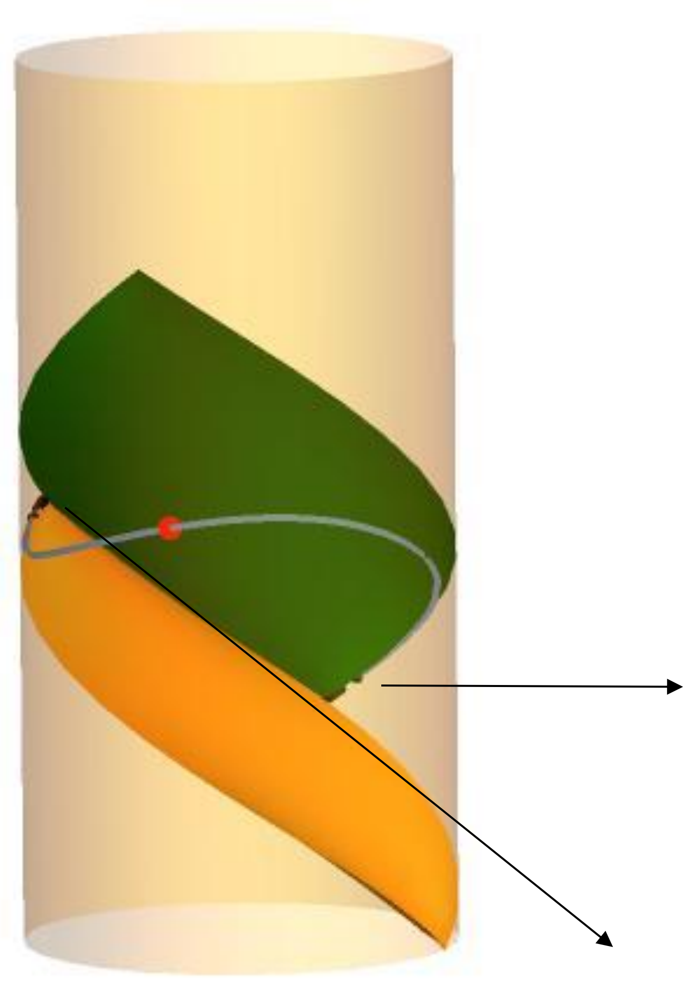
$\Sigma = 0$ Direct light front

$\Sigma = 2$ Bounced light front

Periodic identification – Method of Images



$$G^{\mathrm{BTZ}}_{\mathrm{ret}} = \sum_{n=-\infty}^{\infty} G^{\mathrm{AdS}_3}_{\mathrm{ret}}\big|_{\Phi' \to \Phi' + 2\pi n}$$

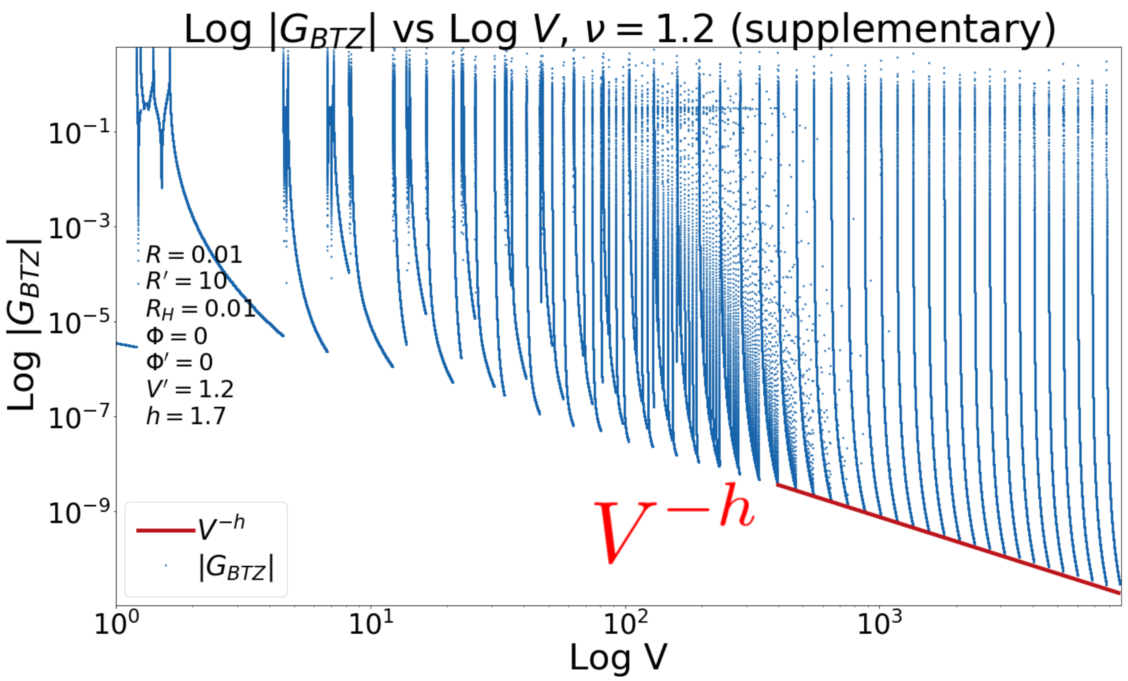
Accumulation of 'images' at ends of 'conformal bifurcation line'

These images emit wave fronts co-rotating with the black hole (orbiting n times)

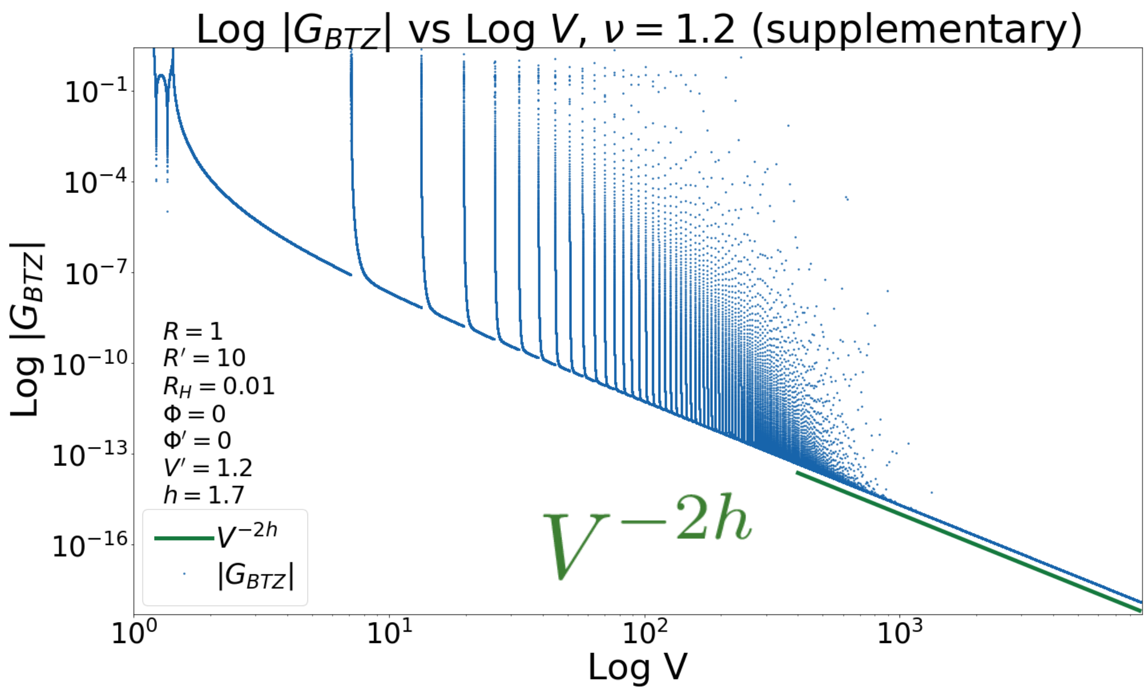
These images emit counter-rotating wave fronts that give rise to the instability

Green function for source off the horizon (non-discrete)

Field point on Horizon



Field point off Horizon



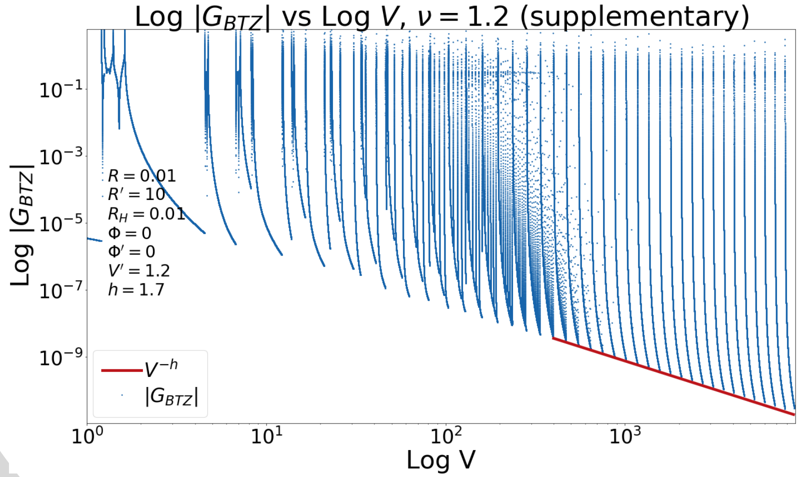
Decay rate differs on and off horizon – indicate instability

$$h = \frac{1}{2} + \nu$$

Decay rate read from envelope indicated

What we've shown

- Recovered horizon instability from the exact retarded Green function in BTZ
- New interpretation – counter-rotating null geodesics cause the instability



Going forward

- Discrete case and on horizon data
- Generalize conclusions beyond just BTZ?
- Connect with other techniques
- Holographic signature?