The Aretakis instability of extreme asymptotically AdS black holes

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Outline

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What is the Aretakis instability?

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What is the Aretakis instability?

Some open questions

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Holographic signature of the instability

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Holographic signature of the instability

Non-compact horizons

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BTZ

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Black Hole stability

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Do initially small perturbations of a (Kerr) black hole evolve under Einstein's vacuum equations to a nearby member of the (Kerr) black hole family of solutions?

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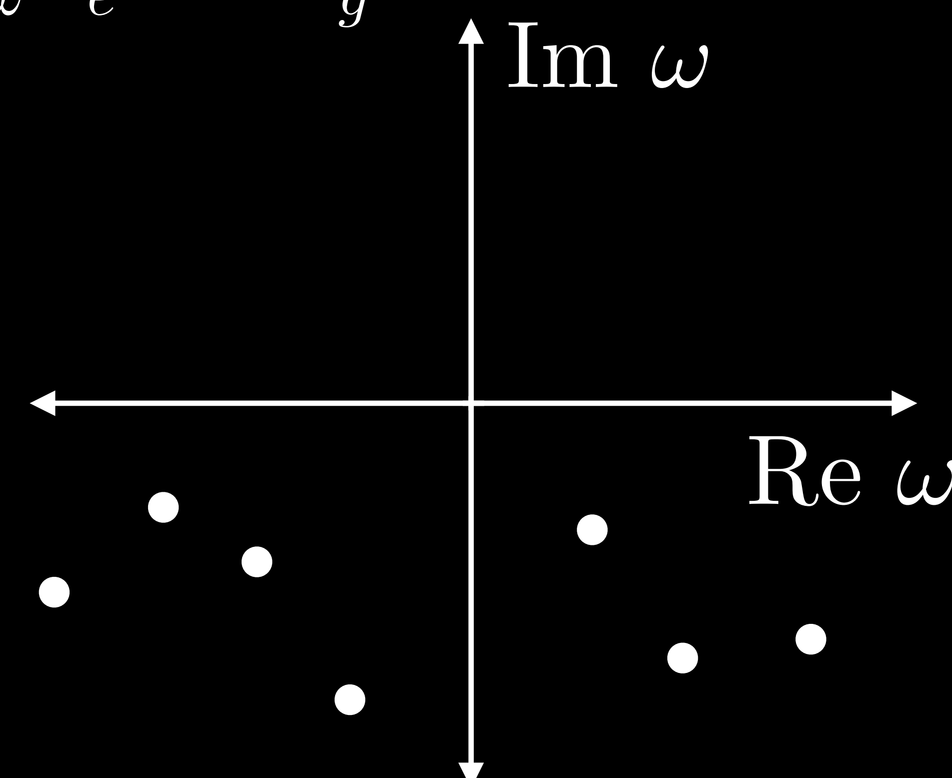
Mode stability $(\Box - \mu^2)G = \delta$ $G = \int d\omega \quad e^{-i\omega(t-t')} g$



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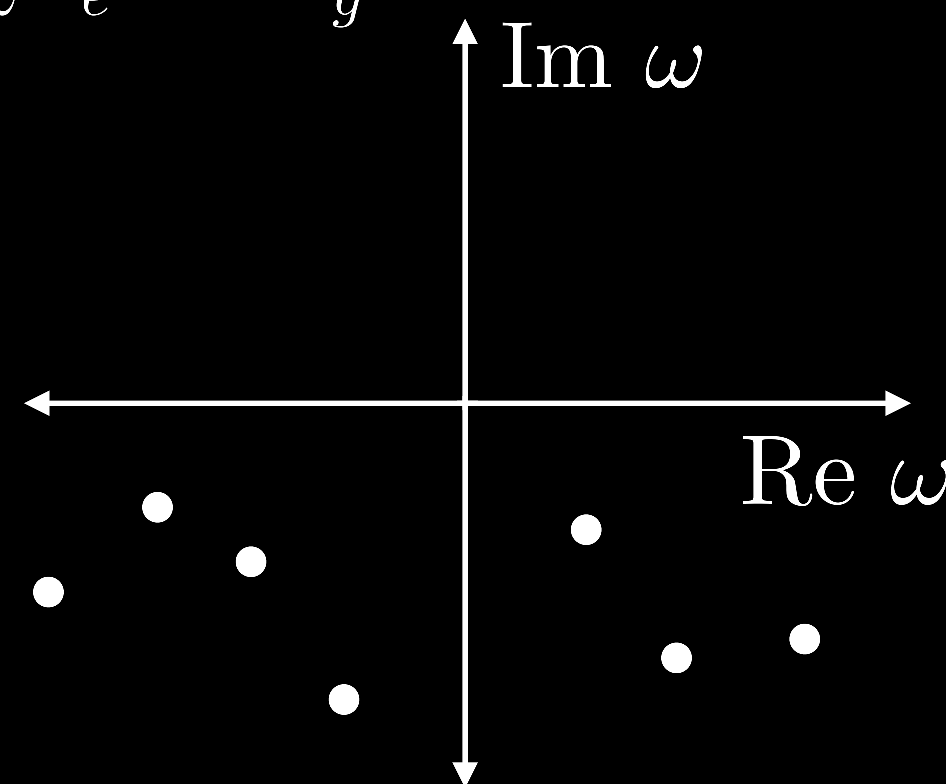

Linear stability

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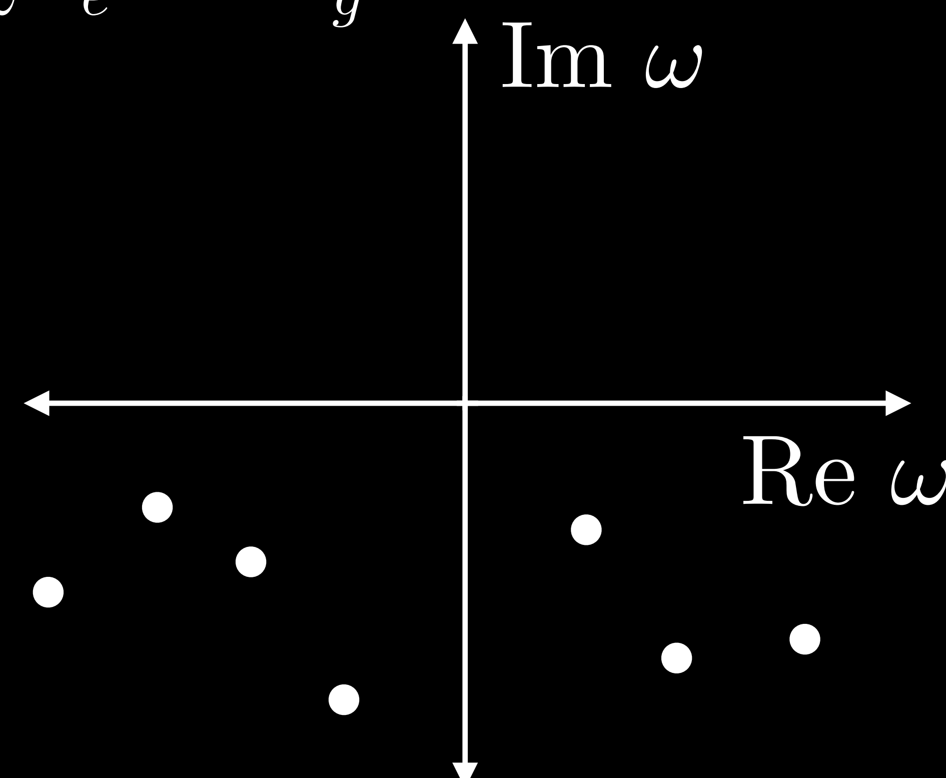

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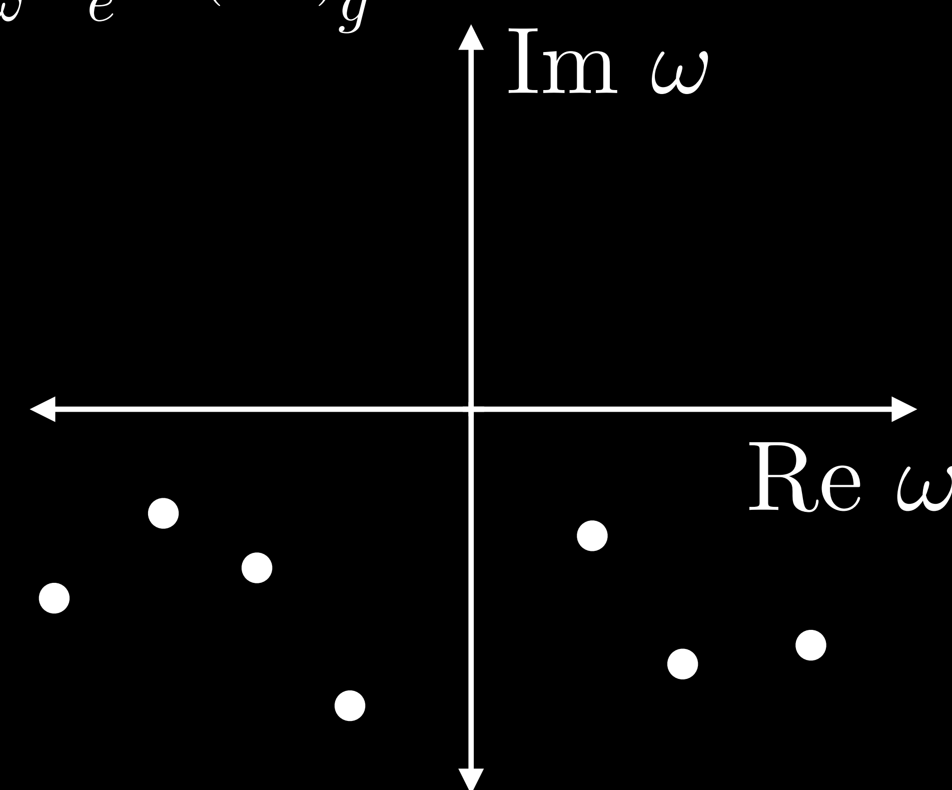

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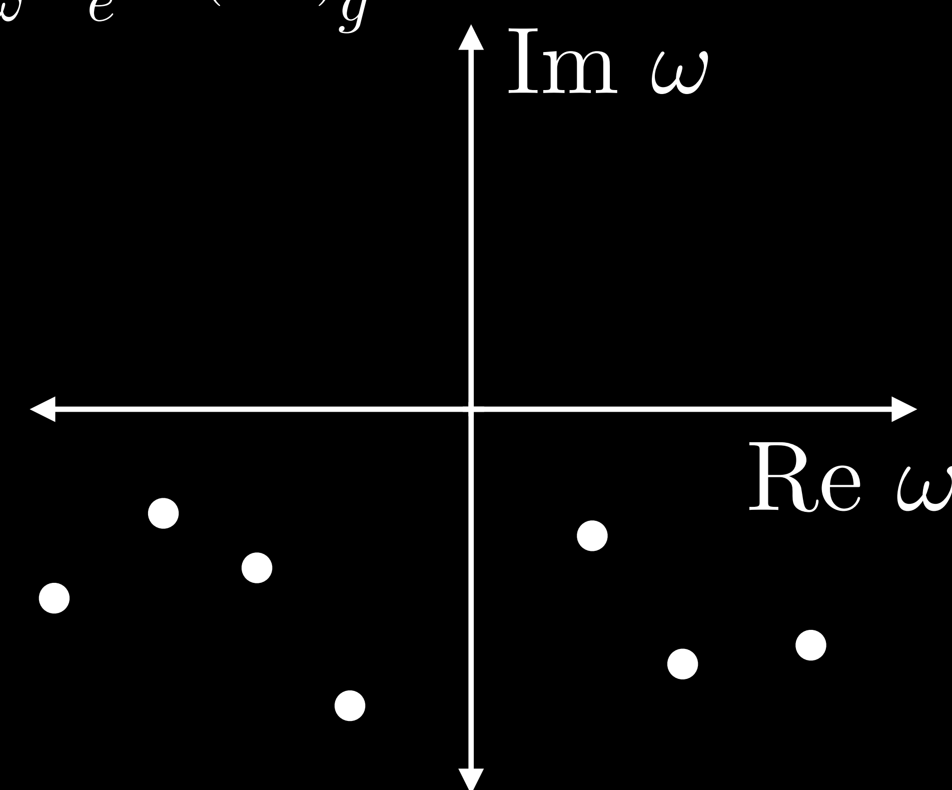

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Aretakis!

Aretakis instability

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Axisymmetric/uncoupled massless scalar field perturbations to extreme Kerr/Reissner-Nordström (Aretakis 2012)

spinning BH charged BH

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along radial direction

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Conservation law on the horizon

Depends only on ***local geometry of horizon*** and not global geometry

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Other fields?

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Scalar, electromagnetic, gravitational perturbations

(Lucietti, Murata, Reall, Tanahashi 2013)

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General extreme horizons

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General extreme horizons	Compact horizon topology (Lucietti, Reall 2012)

Understanding today

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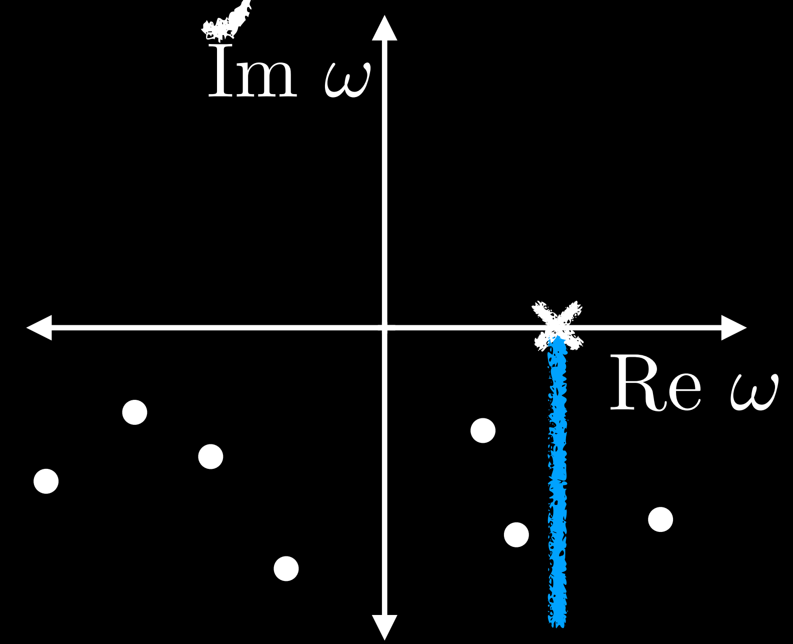
Branch point on complex ω plane



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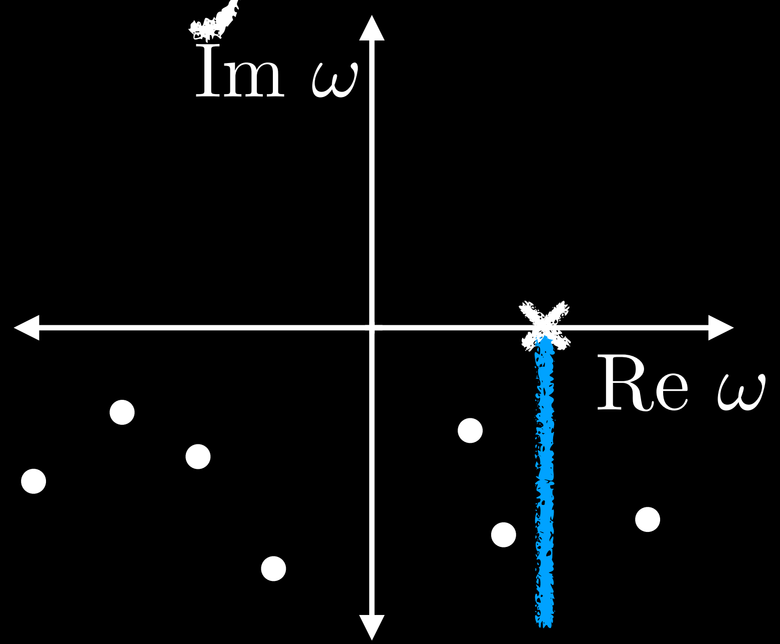

$$\psi \sim v^{-h} f_0(vx) \qquad x \to 0 \ , \quad v \to \infty$$

Understanding today

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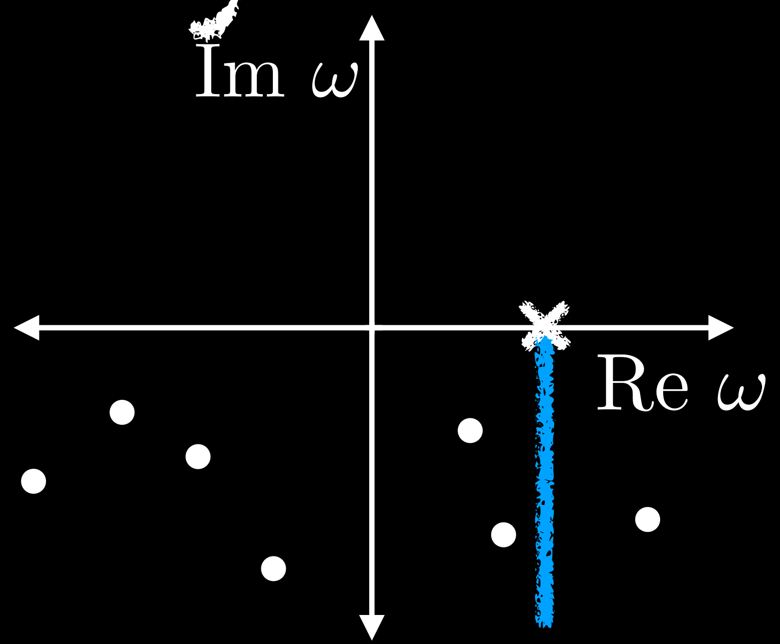

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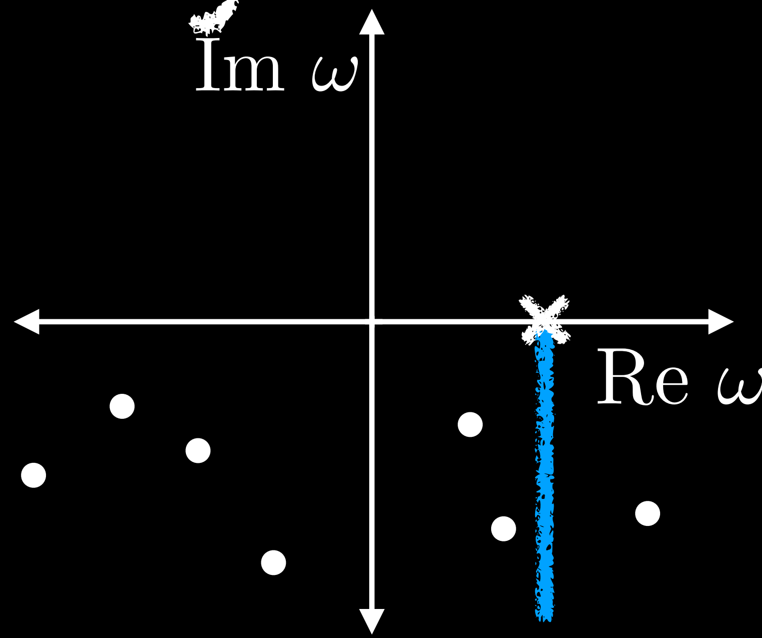

Radial coordinate

Ingoing time

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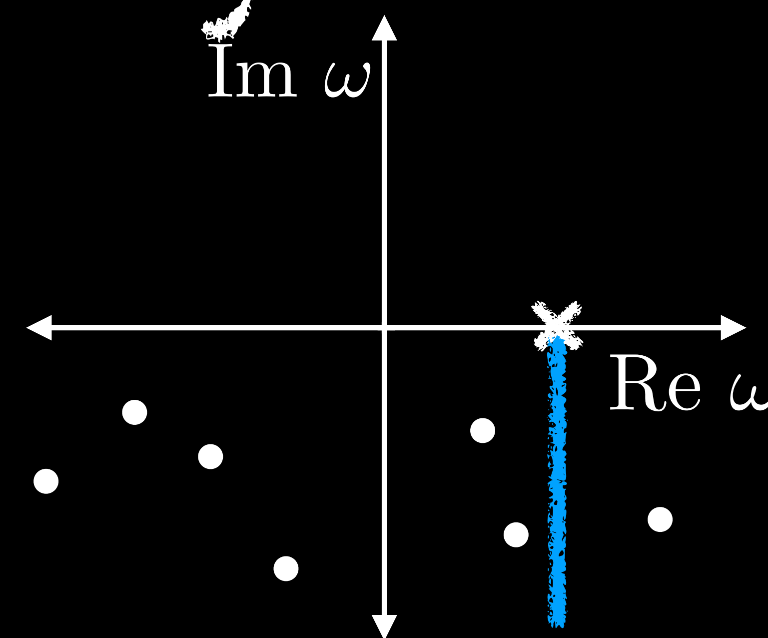

Radial coordinate　　Ingoing time

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Horizon

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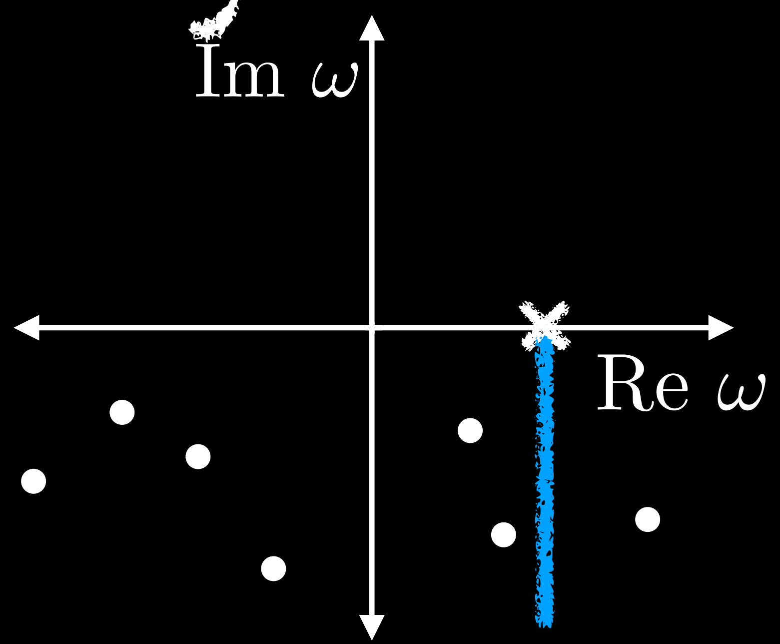

Radial coordinate ↑ , Ingoing time ↑

$$\psi \sim v^{-h} f_0(vx) \qquad x \to 0\,, \quad v \to \infty$$

Horizon ↓ , Late times ↓

Understanding today

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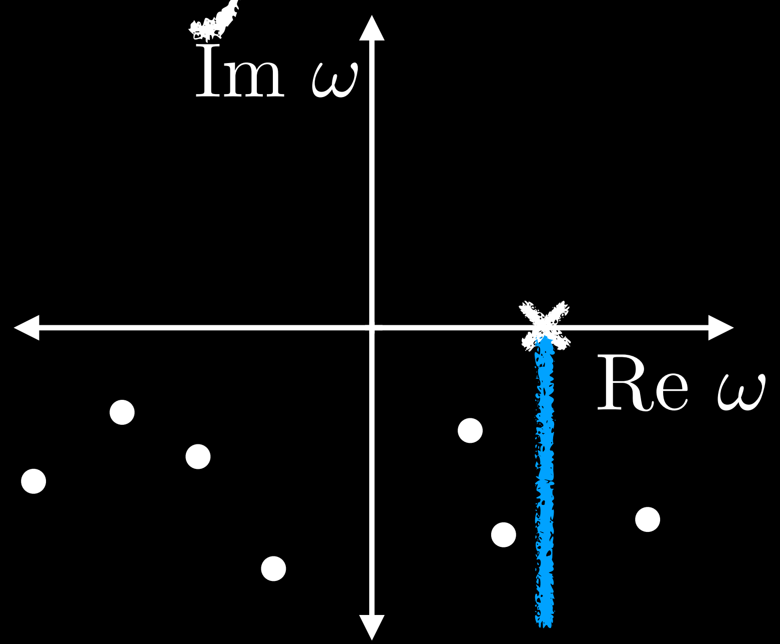

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$$\psi \sim v^{-h} f_0(vx) \qquad x \to 0 \,, \quad v \to \infty$$

Radial coordinate (x), Ingoing time (v)

Horizon ($x \to 0$), Late times ($v \to \infty$)

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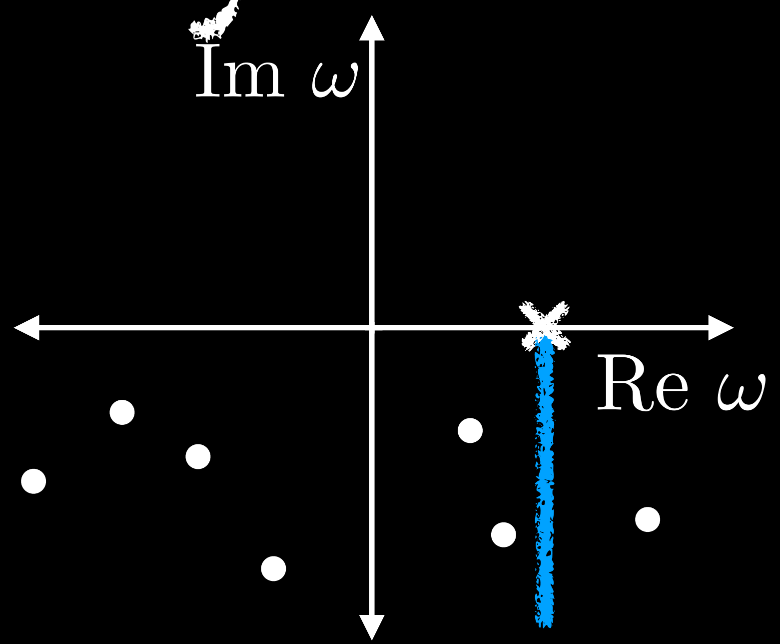

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$$\mathrm{Re}\ h \geq 1/2$$

$$h \equiv h(\mu, q)$$

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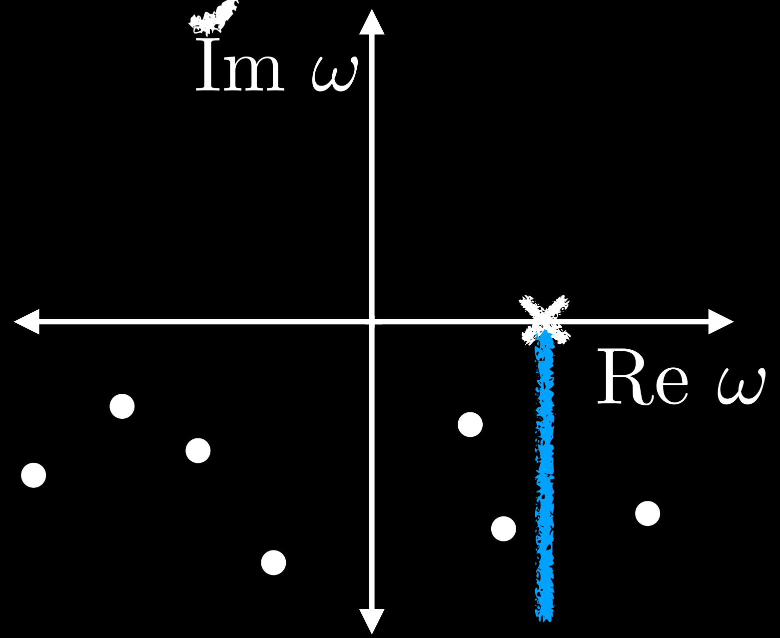

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Horizon — Late times

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$$\partial_x^n \psi \sim v^{-h+n} f_1(vx) \qquad x \to 0\,, \quad v \to \infty$$

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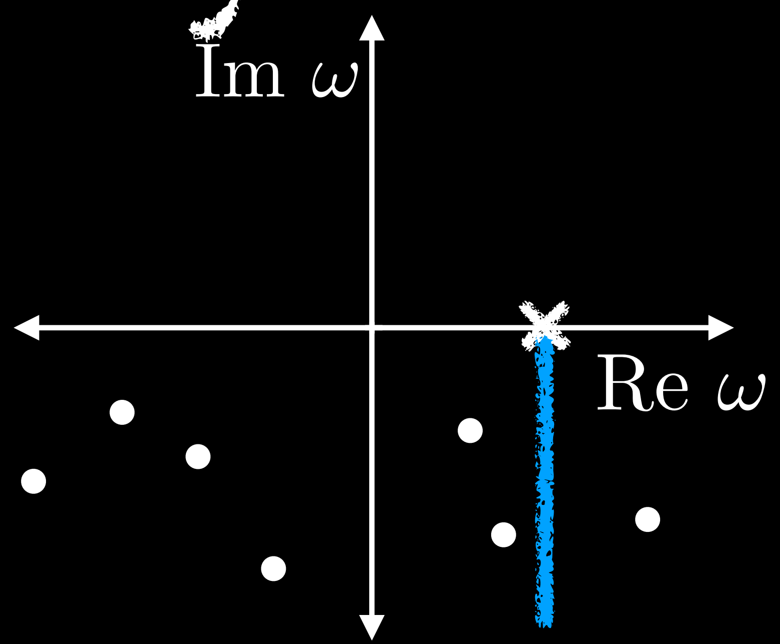

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On Horizon decay rate

$$v^{-h}$$

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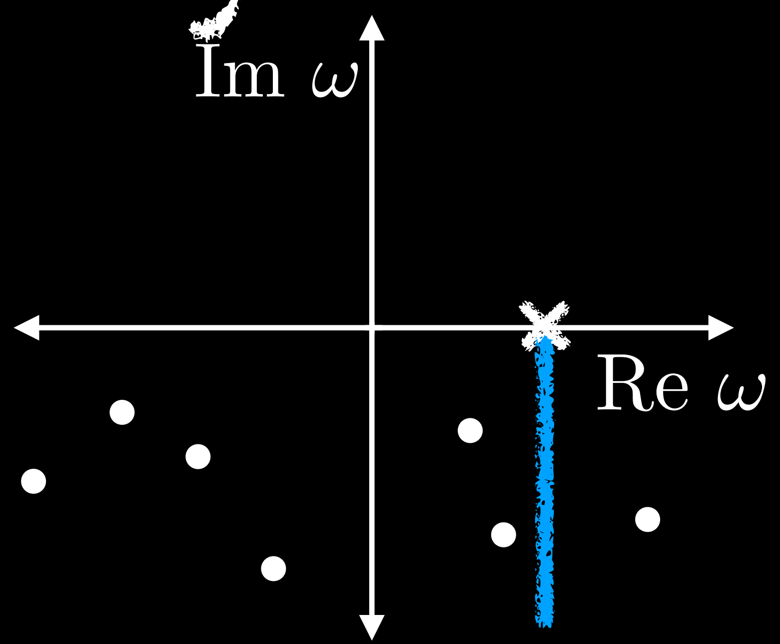

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Off Horizon decay rate

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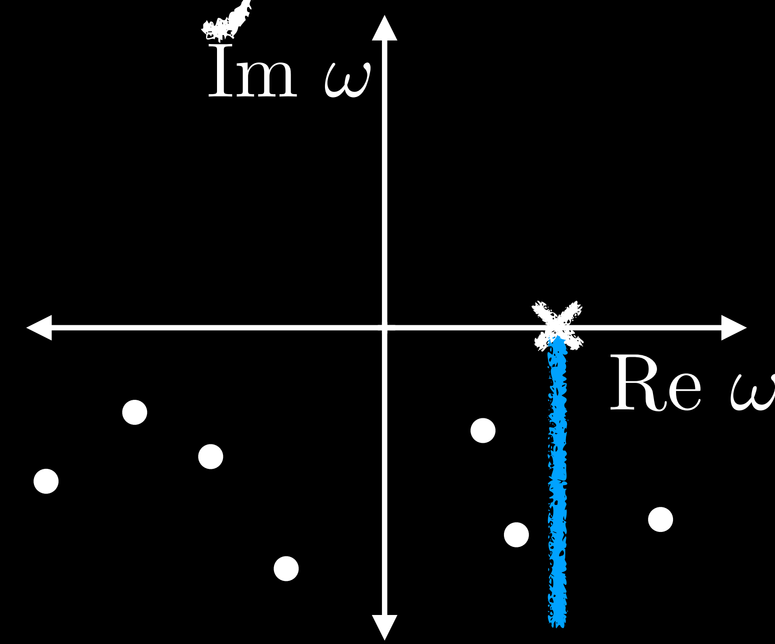

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Not typical instability - scalar field itself decays

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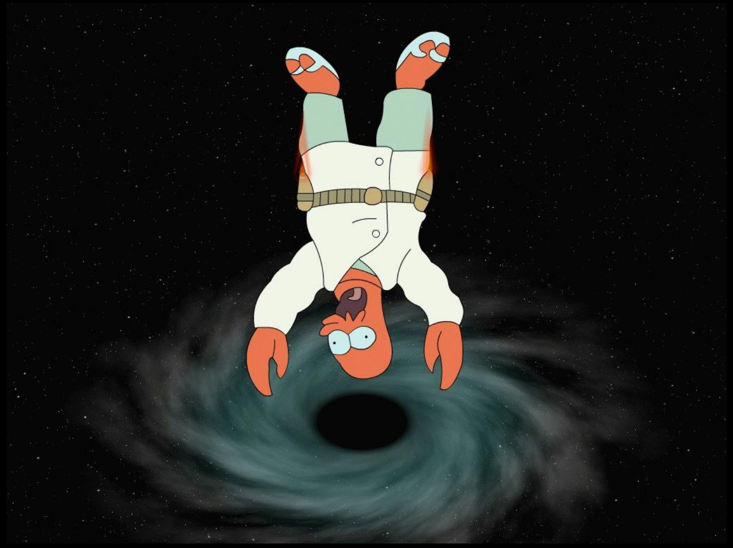

Only Ingoing observers on the horizon see this effect

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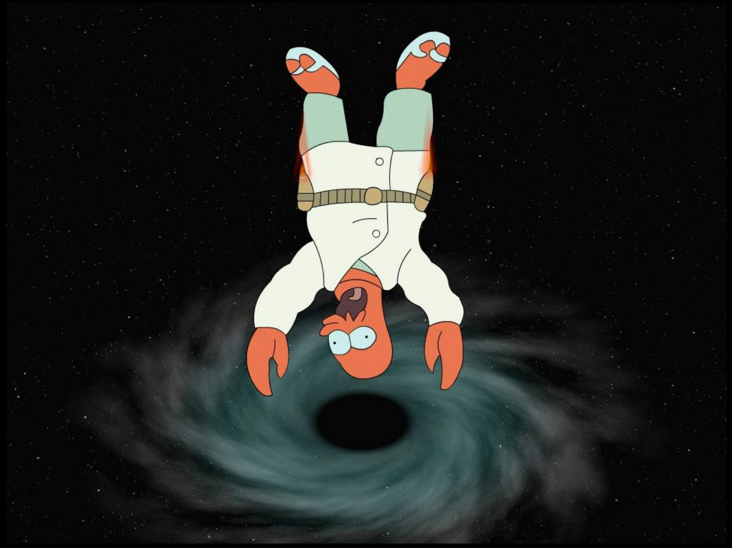
General covariance prevents observer independent quantities from becoming large



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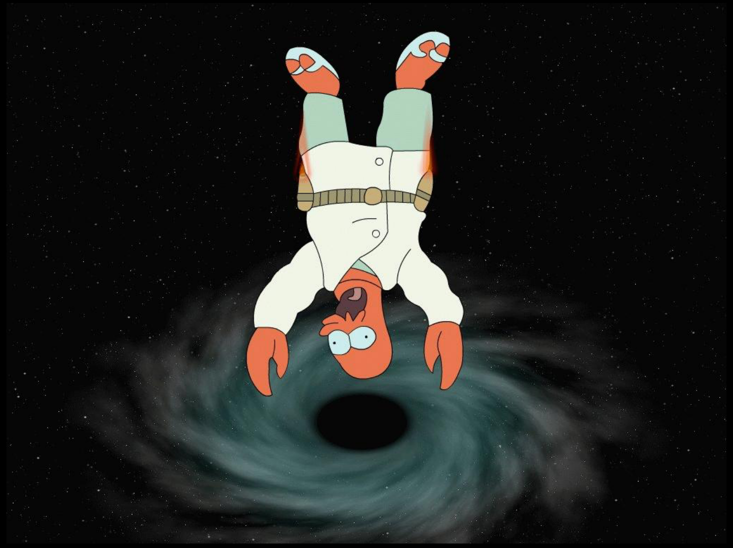

Field strength invariant

$$F^{\alpha\beta}F_{\alpha\beta}$$

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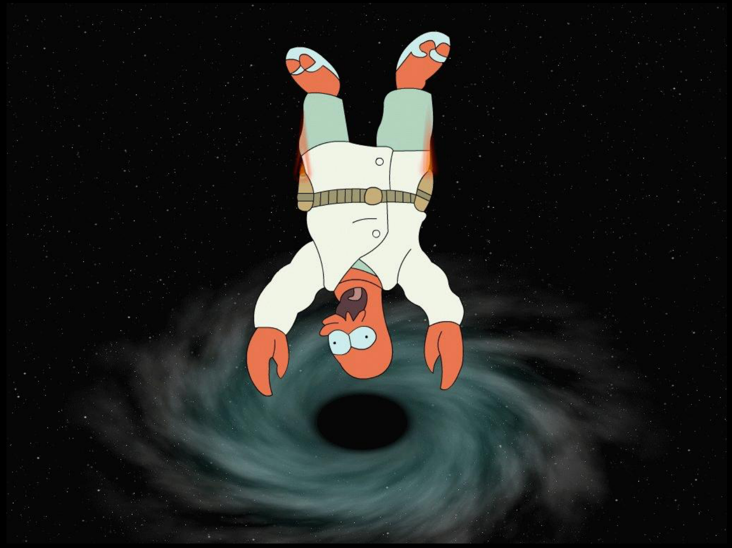

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Field strength invariant	Kretschmann invariant
$F^{\alpha\beta}F_{\alpha\beta}$	$R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$
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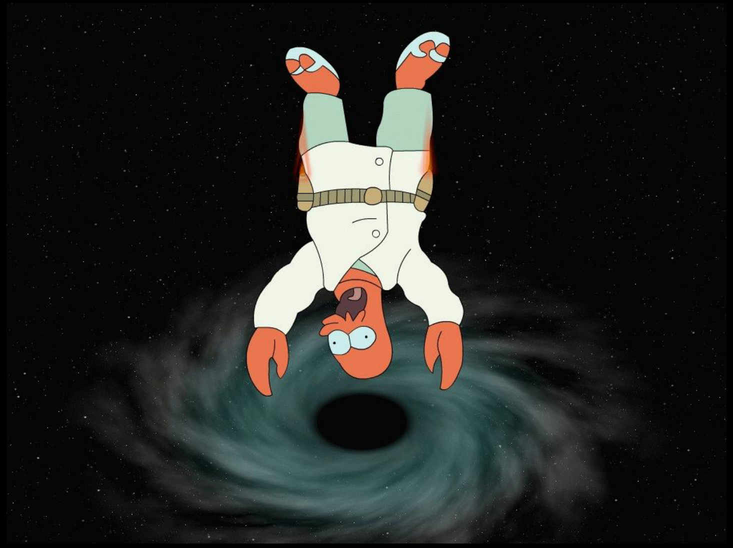

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Field strength invariant	Kretschmann invariant	Squared electric field strength observed by infalling observer
$F^{\alpha\beta}F_{\alpha\beta}$	$R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$	$E^2 = F_{\mu\alpha}u^\alpha F^{\mu\beta}u_\beta$
decays	decays	grows

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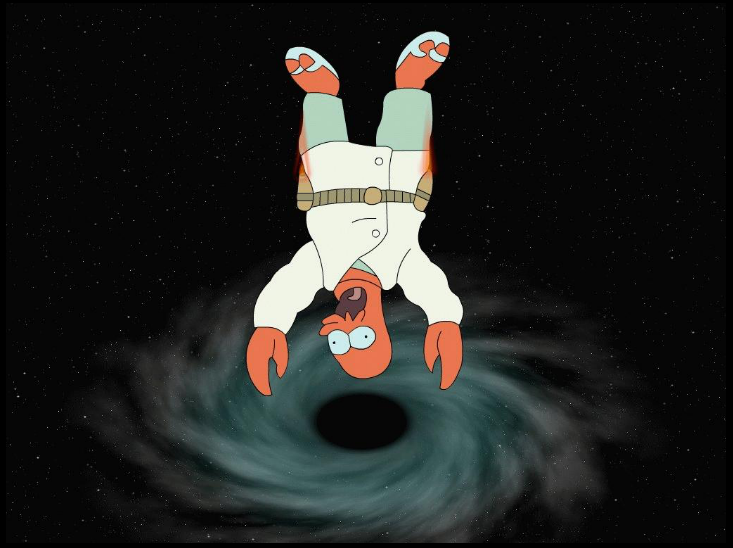

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Open questions

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Theorem ensures horizon instability for extremal horizons with compact topology

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No explicit example with non-compact horizon topology studied

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Holographic signature to the horizon instability of asymptotically AdS black holes?

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2 Non-overlapping techniques

Unify?

Conserved quantity on extremal horizons

Initial data must extend to the horizon

`Discrete' case only (eg. massless, axisymmetric scalar in Kerr)

Aretakis, Lucietti, Murata, Reall, Tanahashi, Virmani…

Mode sum approach using matched asymptotics

Initial data is supported entirely outside the horizon

`Non-discrete' case only generally (eg. non-axisymmetric scalar in Kerr)

Casals, Gralla, AR, A.Zimmerman, P.Zimmerman

pRNAdS_5

Planar Reissner-Nordström
Anti-de-Sitter in 5*d*

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Maxwell field coupled to AdS gravity in 5 dimensions

pRNAdS_5

Planar Reissner-Nordström
Anti-de-Sitter in 5*d*

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Maxwell field coupled to AdS gravity in 5 dimensions

$$ds^2 = \frac{\ell^2}{z^2}\left(-f\,d\tau^2 + \frac{dz^2}{f} + d\vec{y}^2\right)$$

$$f = 1 - 3z^4(1-2\sigma) + 2z^6(1-3\sigma)$$

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Parameters of black hole - charge density and AdS length

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Re-parameterize to make temperature a parameter

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Re-parameterize to make temperature a parameter

$$T \propto \sigma \to 0$$

Extremal limit

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Re-parameterize to make temperature a parameter

$$T \propto \sigma \to 0$$

Extremal limit

Boundary

$$z \to 0$$

Horizon

$$z \to 1$$

$$1 - z \sim x \qquad x \ll 1$$

Framework of Calculations

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$$(D^2 - \mu^2)\psi = 0$$

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details of geometry

$$D^\mu = \nabla^\mu - iqA^\mu$$

coupled to gauge field

Framework of Calculations

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$$(D^2 - \mu^2)G = \delta$$

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Ansatz
$$G = \int \frac{d\omega d^3k}{(2\pi)^4} e^{-i\omega t + i\vec{k}.\vec{y}} g(\vec{k}, \omega, x)$$

Framework of Calculations

$$(D^2 - \mu^2)\psi = 0$$

details of geometry

$$D^\mu = \nabla^\mu - iqA^\mu$$

coupled to gauge field

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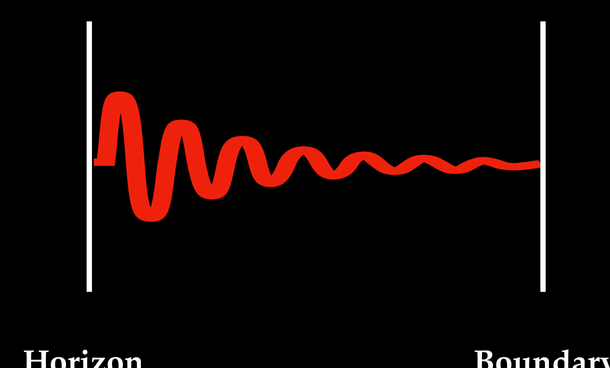
ODE for g

Ingoing waves on horizon

$$R_{\text{in}} \sim e^{-i\omega r_*} \qquad x \to 0$$



Appropriate decay at boundary of AdS

$$R_{\text{far}} \sim x^{-\Delta_+} \qquad x \to \infty$$

Framework of Calculations

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Matched asymptotic expansions

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Near region $1 - z \ll 1$

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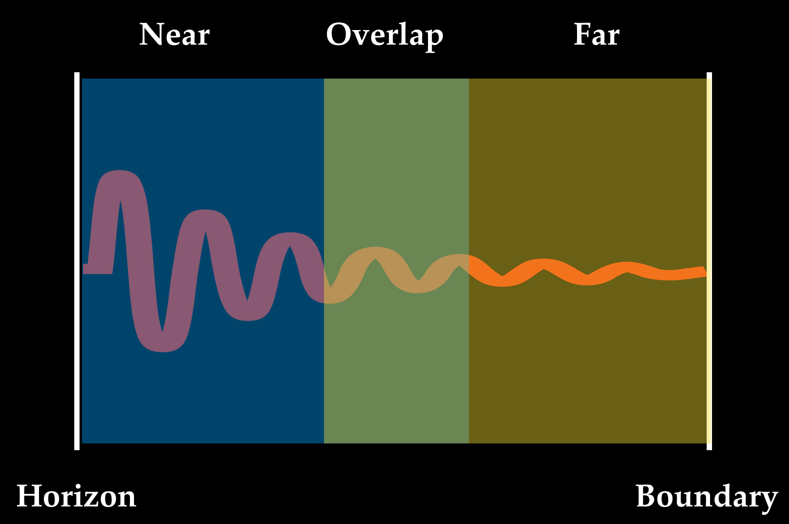

$\omega \ll 1$	Late times
Near region	$1 - z \ll 1$
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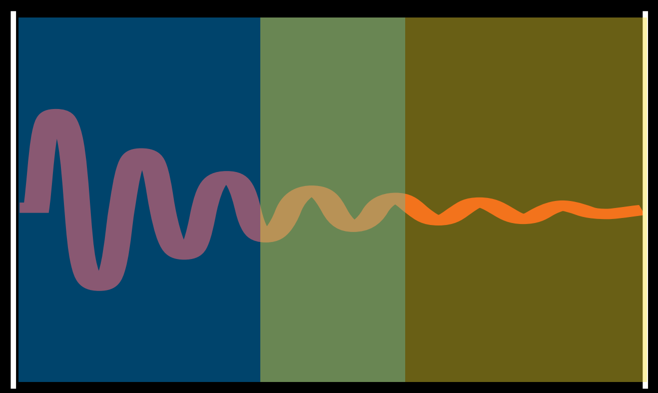

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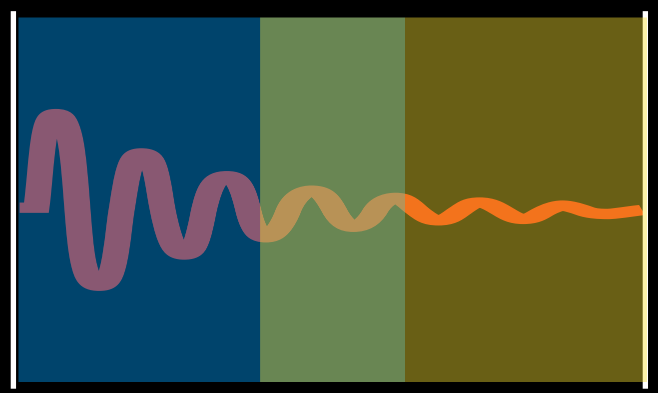

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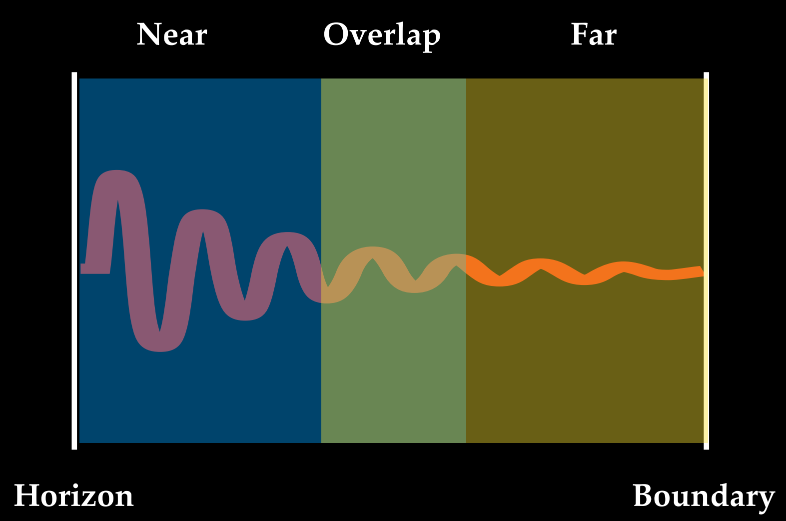

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$$G_k \equiv L^{-1}[g] \sim v^{-h} f(vx, \vec{k}) \qquad v \to \infty \ , \ x \to 0$$

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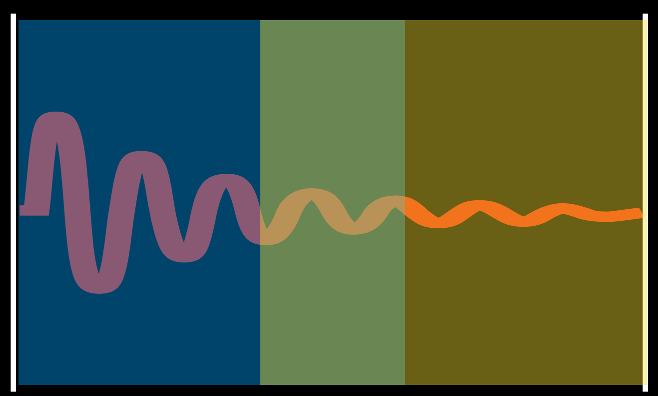

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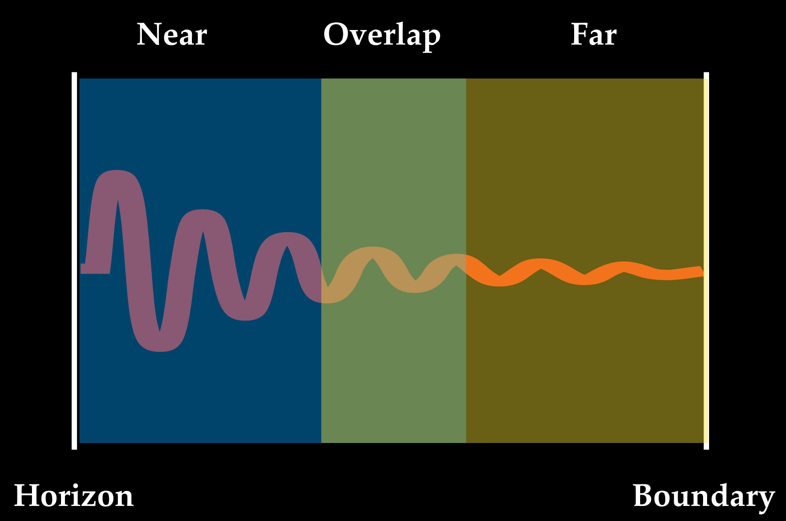

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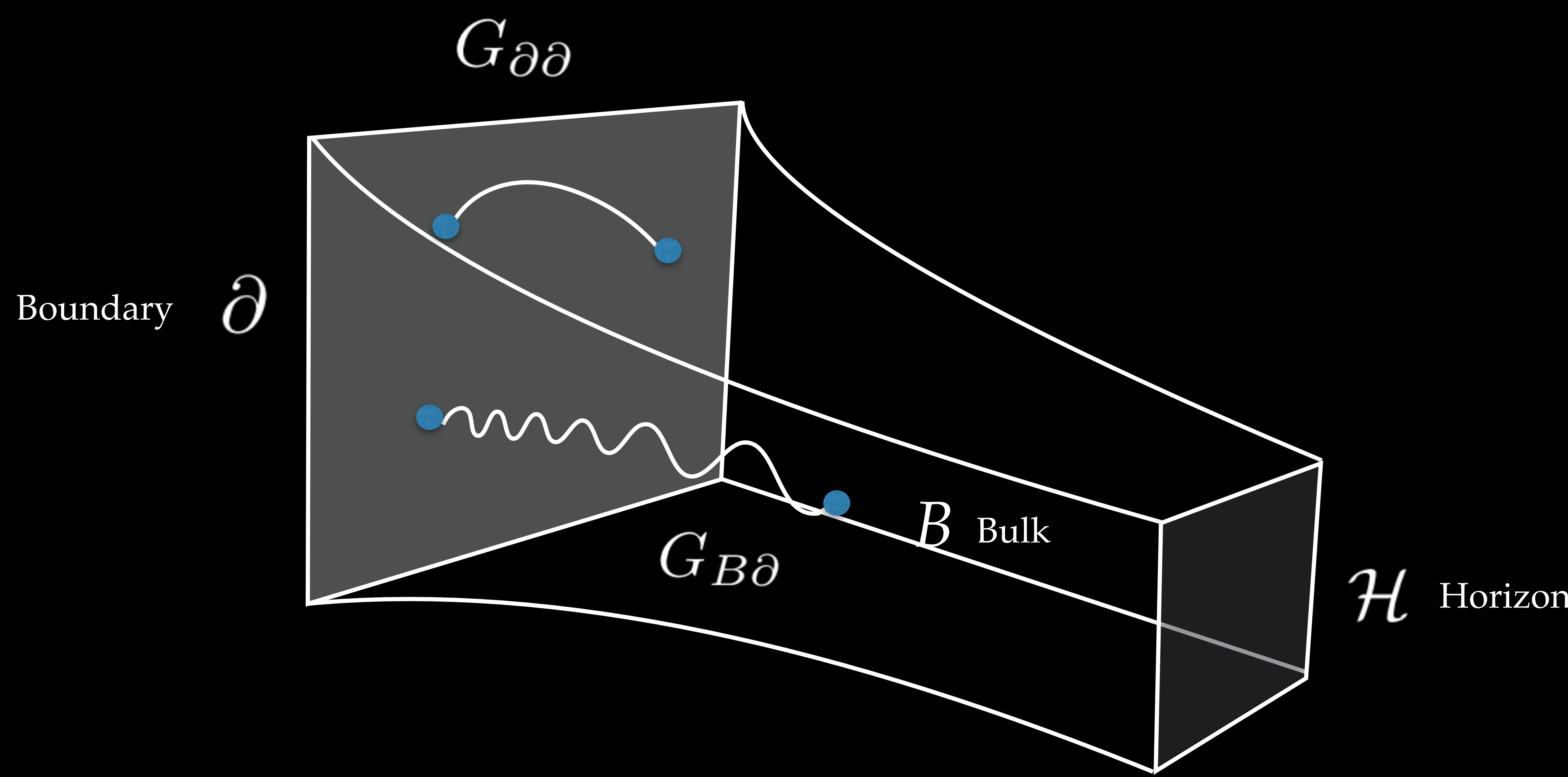
$$G = \int d^3k \quad G_k$$ if tractable

Boundary propagators

Boundary propagators



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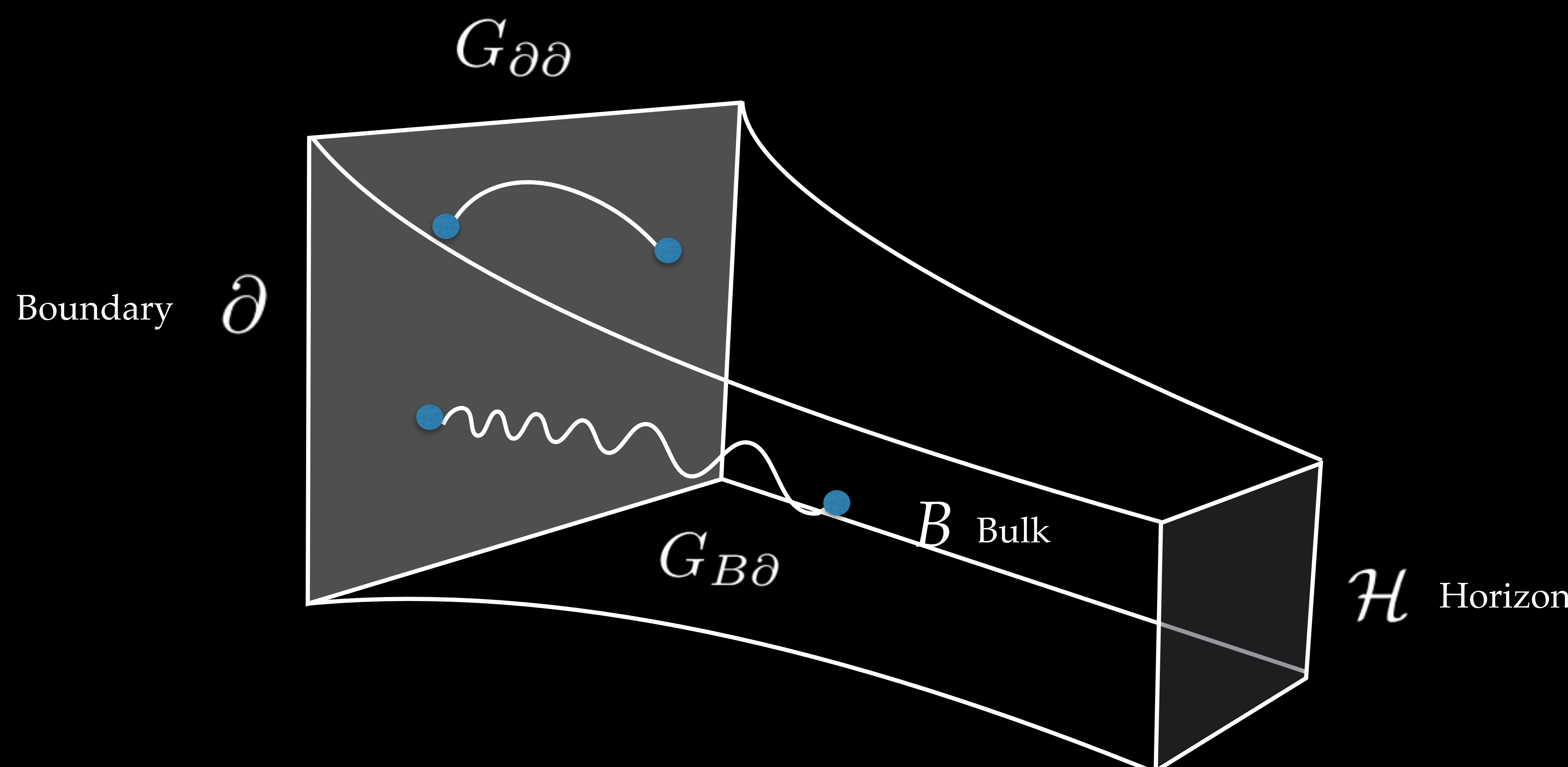
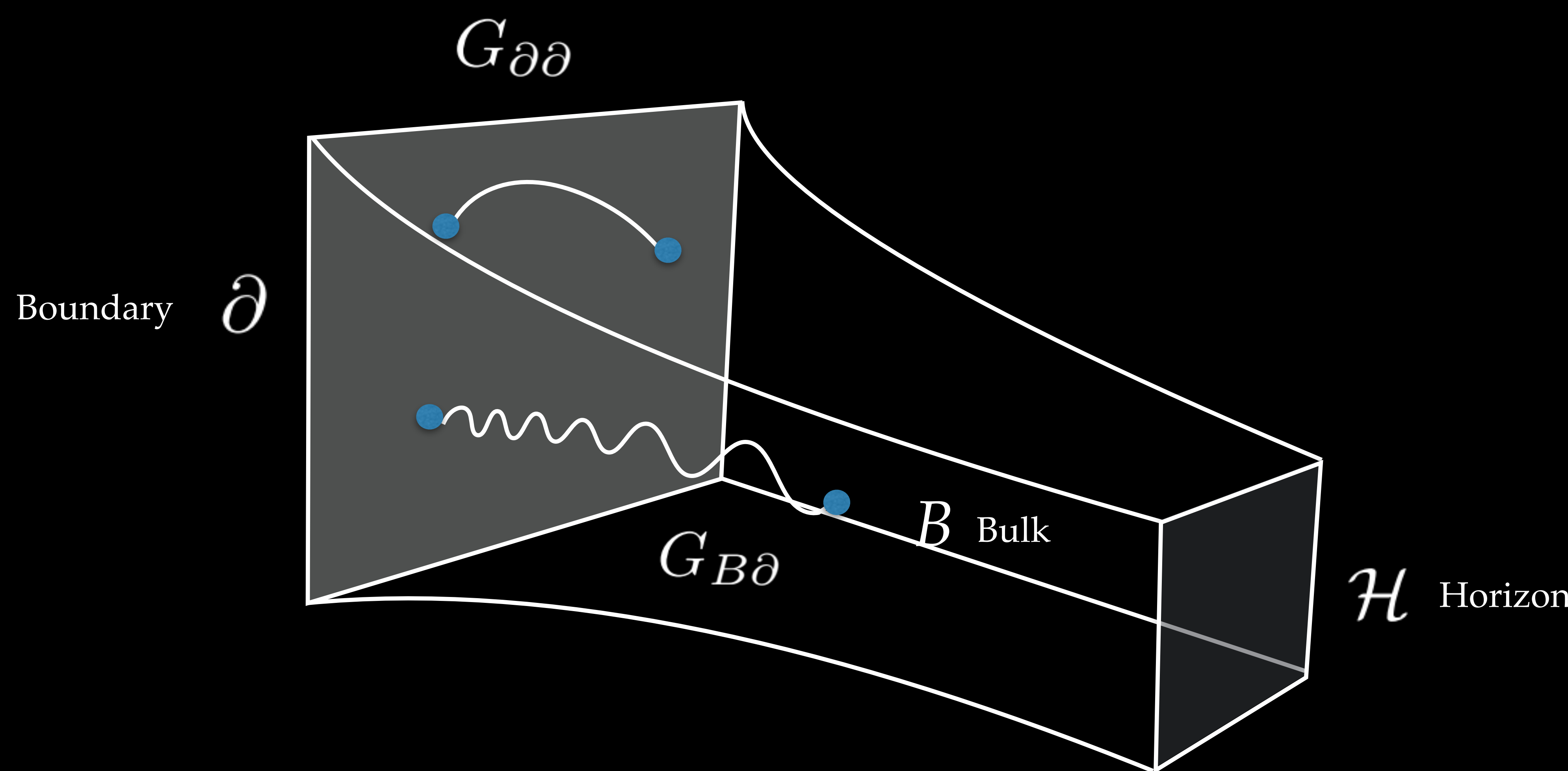


$$G_{\partial B}(\tau, y, z) = \ell^{3/2} \lim_{z' \to 0} (z')^{-\Delta_+} G$$

Bulk-Boundary propagator

Boundary propagators



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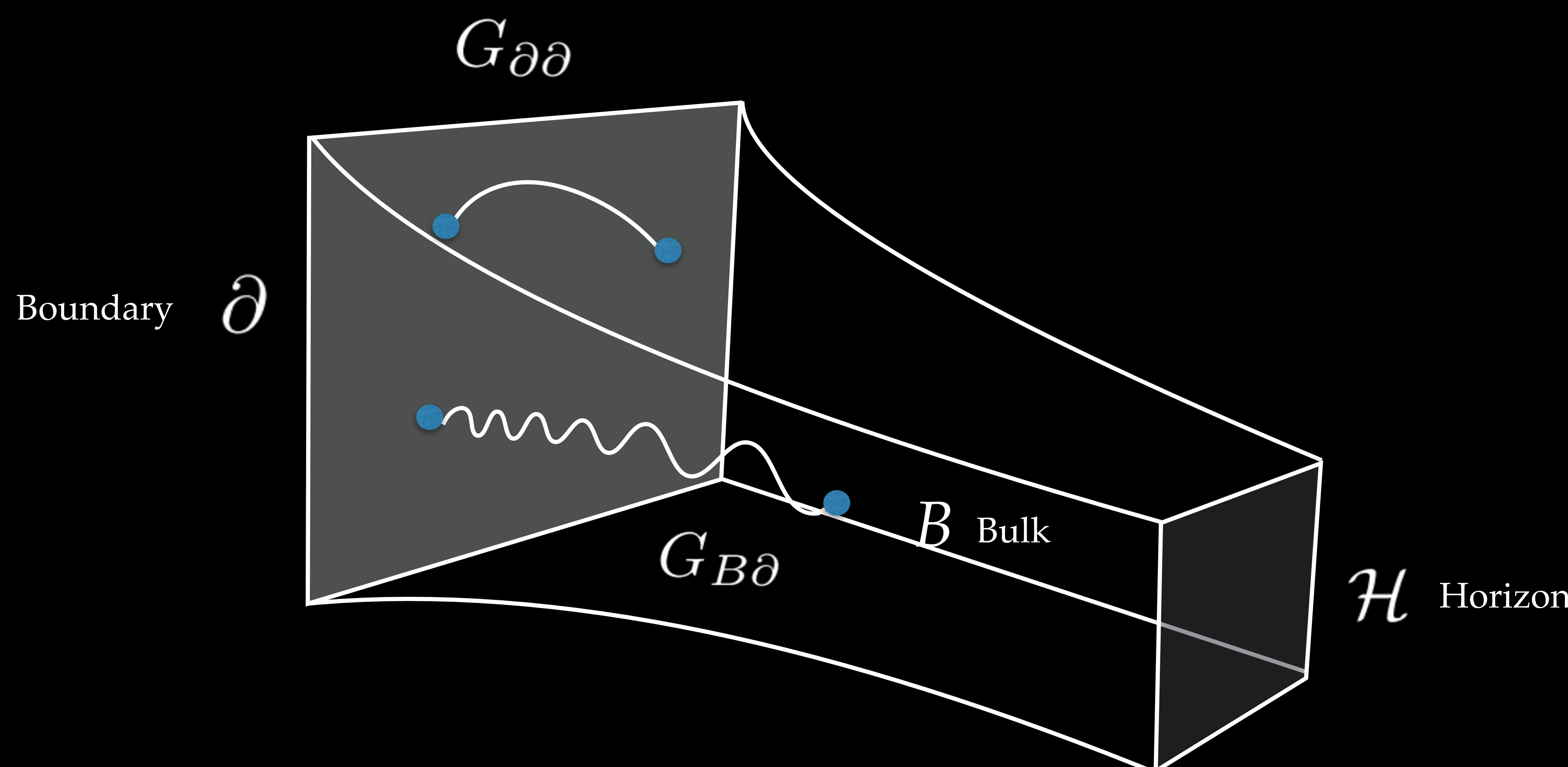

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Boundary-Boundary propagator

where

$$\Delta_\pm = 2 \pm \sqrt{4 + \ell^2 \mu^2}$$

Results

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$$G_{\partial B}^{\text{near}} \sim \frac{\mathcal{C}}{\ell^{3/2} y} \left(\frac{k_c}{y}\right)^{3/2} e^{-yk_c} (6v)^{-1/2-i\hat{e}} (1+6v(1-z))^{-1/2+i\hat{e}}, \qquad y \to \infty$$

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What is the CFT dual to the Aretakis instability?

CFT dual?

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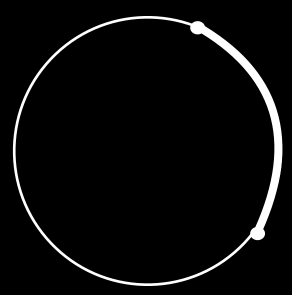
Don't see anything in a 2 point function on the boundary



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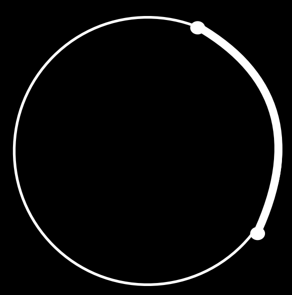

The Aretakis instability is seen only on the horizon of the black hole

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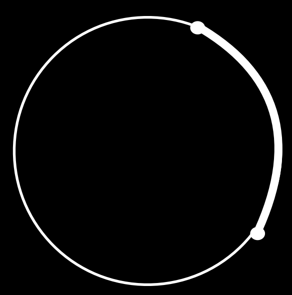

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On Horizon decay rate

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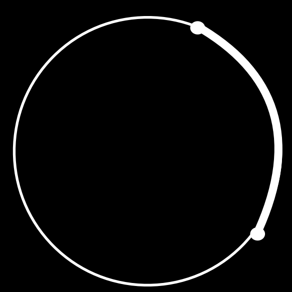

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On Horizon decay rate	Off Horizon decay rate
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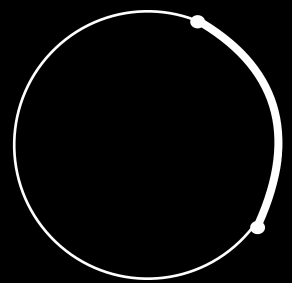

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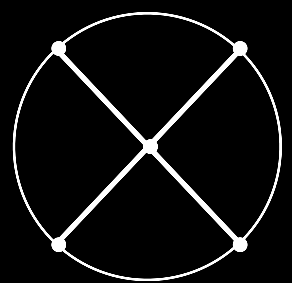
Interacting theory - integrate through the bulk to include near-horizon effects



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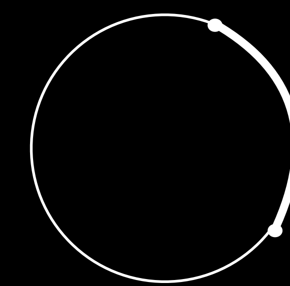

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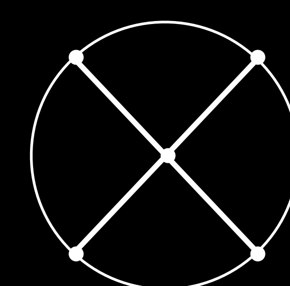

Do we see a signature for the Aretakis instability for a CFT that lives on the boundary?

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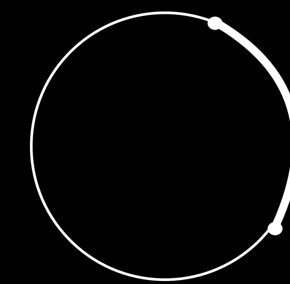

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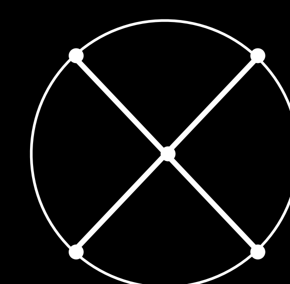

Do we see a signature for the Aretakis instability for a CFT that lives on the boundary?

Boundary correlators grow or decay slower than expected?

Scaling arguments

Scaling arguments

Temporal conformal transformations

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$$t \to \lambda^{-1} t, \quad \mathcal{O} \to \lambda^{1/2} \mathcal{O}$$

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At tree level,

$$\langle \mathcal{O}_1 \mathcal{O}_2 \rangle \to \lambda \langle \mathcal{O}_1 \mathcal{O}_2 \rangle \qquad \langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle \to \lambda^2 \langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle \quad \text{under} \quad t_i \to \lambda^{-1} t_i$$

late times

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late times

Given that the scaling of $G^{\text{near}}_{\partial B}$ is different, do we see a signature from near-horizon region?

$$G^{\text{near}}_{\partial B} \sim v^{-1/2}, \quad 1 - z \to 0 \ , \ v \to \infty$$

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Temporal conformal symmetry of boundary operator preserved due to the Aretakis instability!

Generalization

Generalization

λ^0 for each near-region vertex

λ^{-1} for each far-region vertex

$\lambda^{1/2}$ for each boundary-near propagator

λ for each boundary-far propagator

λ^0 for each near-near propagator

Generalization

λ^0 for each near-region vertex

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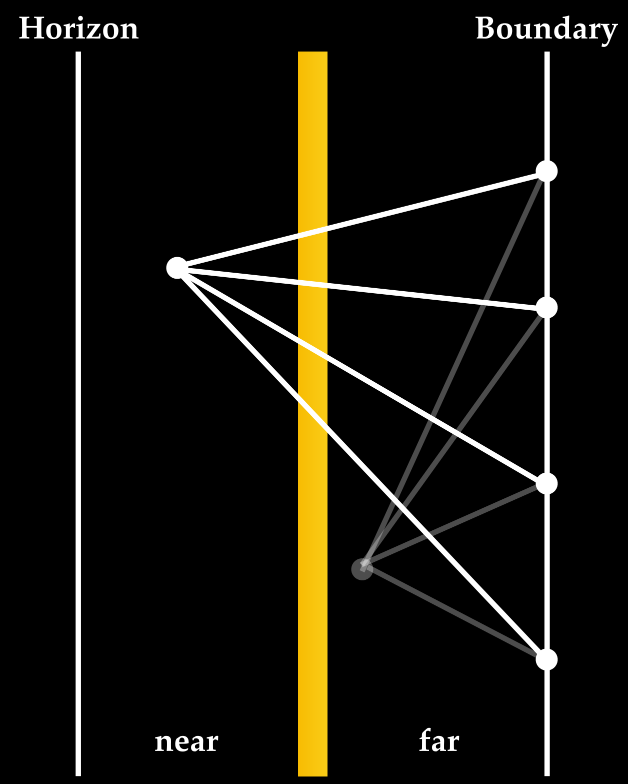
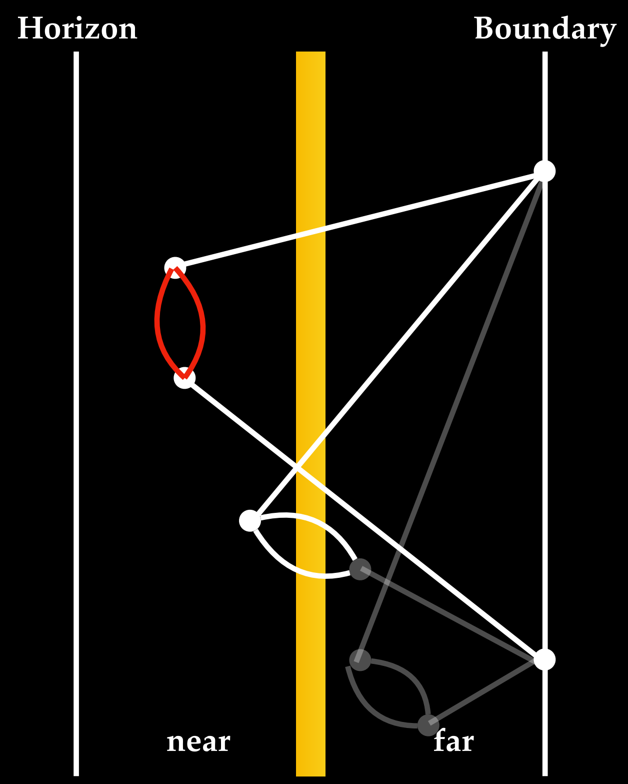


$\Lambda\Phi^4$



$\Lambda\Phi^3$

Summary - RNAdS

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The Aretakis instability persists in spacetime with non-compact horizon topology

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Holographic meaning of Aretakis instability remains a mystery

Open questions

Theorem ensures horizon instability for extremal horizons with compact topology

No explicit example with non-compact horizon topology studied

Holographic signature to the horizon instability of asymptotically AdS black holes?

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Aretakis, Lucietti, Murata, Reall, Tanahashi, Virmani…

Open questions

Theorem ensures horizon instability for extremal horizons with compact topology

No explicit example with non-compact horizon topology studied

Holographic signature to the horizon instability of asymptotically AdS black holes?

2 Non-overlapping techniques

Conserved quantity on extremal horizons

Initial data must extend to the horizon

`Discrete' case only (eg. massless, axisymmetric scalar in Kerr)

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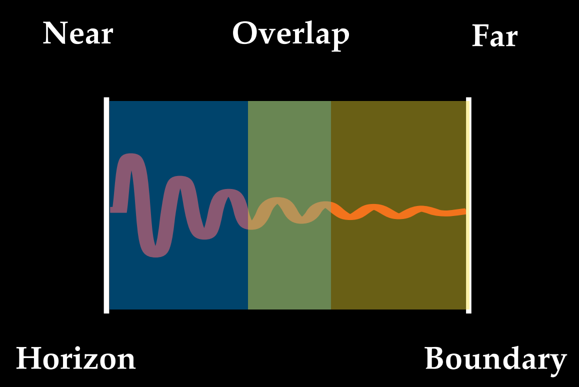

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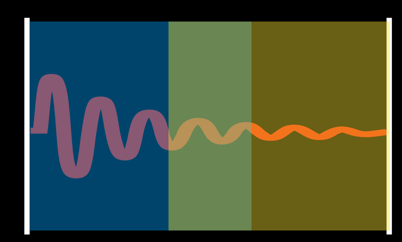

$$\partial_x^x\psi \sim v^{-h+n}f_1(vx)$$

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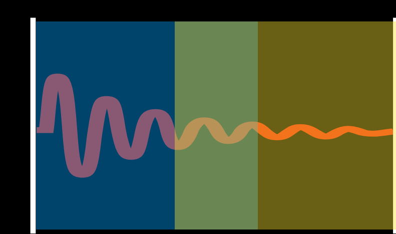

Horizon Boundary

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Each mode

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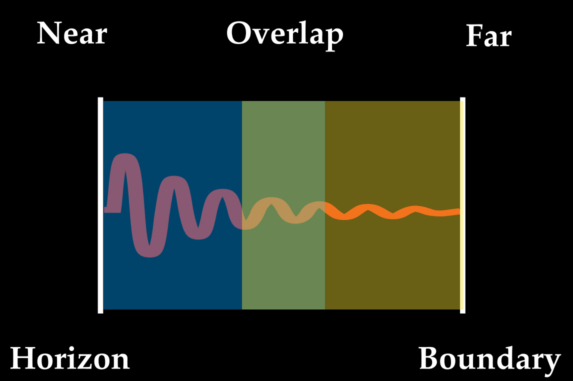

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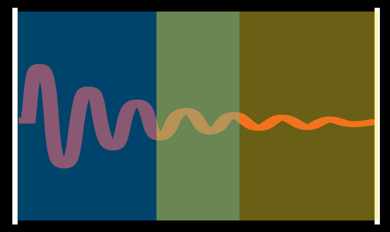

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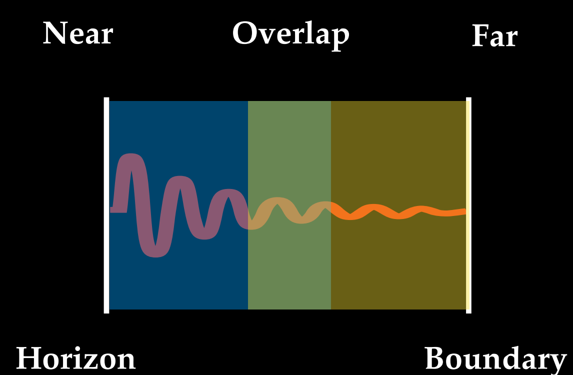

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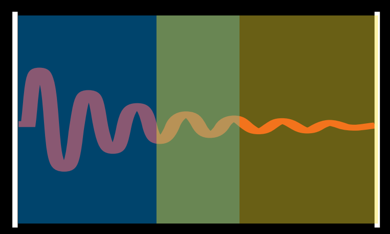

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In the BTZ black hole, we can construct the full Green function allowing us to explore further

Bañados Teitelboim Zanelli (BTZ)

(ongoing)

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Rotating black hole in 3*d* - asymptotically AdS

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Gravity in 3*d* special - All vacuum spacetimes have constant curvature, locally

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Periodically identify along a Killing field - different global geometries

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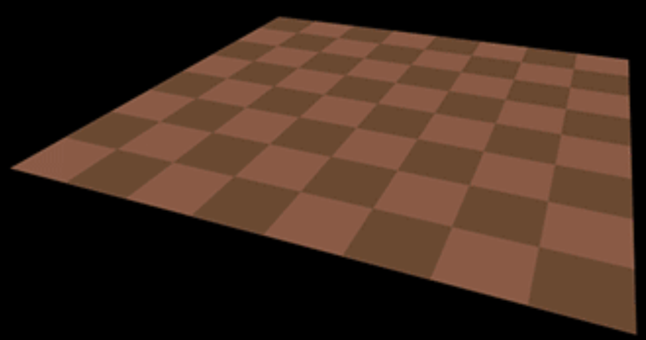

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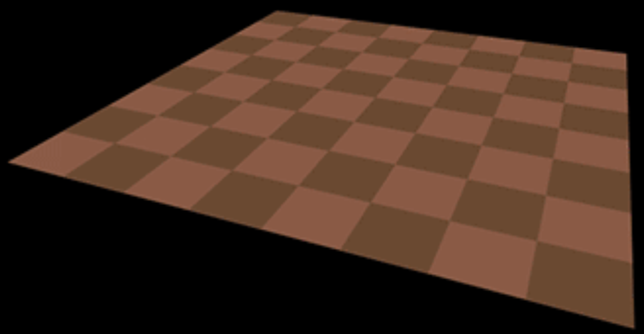

Rotating black hole - similar properties to Kerr

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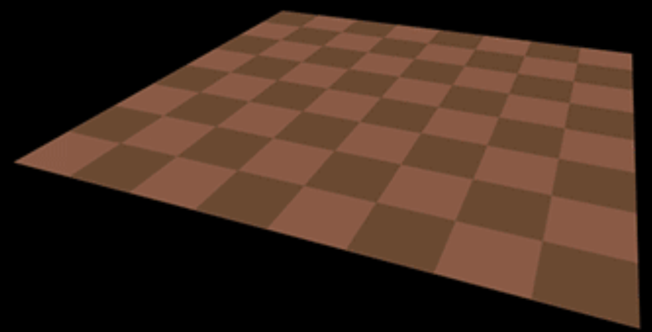

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Can build $G_{\rm ret}^{\rm BTZ}$ from $G_{\rm ret}^{\rm AdS_3}$ (Method of Images)

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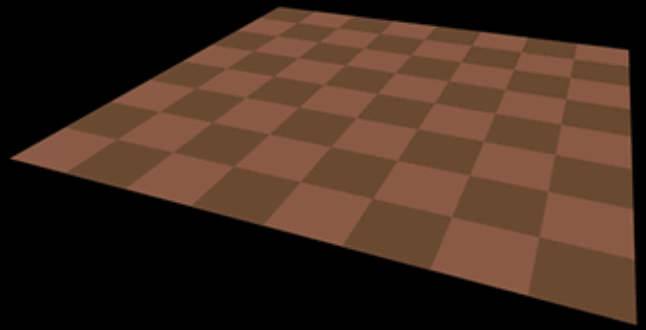

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$$G_{\rm ret}^{\rm BTZ} = \sum_{n=-\infty}^{\infty} G_{\rm ret}^{\rm AdS_3} \,|_{\Phi' \to \Phi' + 2\pi n}$$

(Steif 1993)

Scalar field perturbation $G_{\rm ret}^{\rm AdS_3}$ Green function

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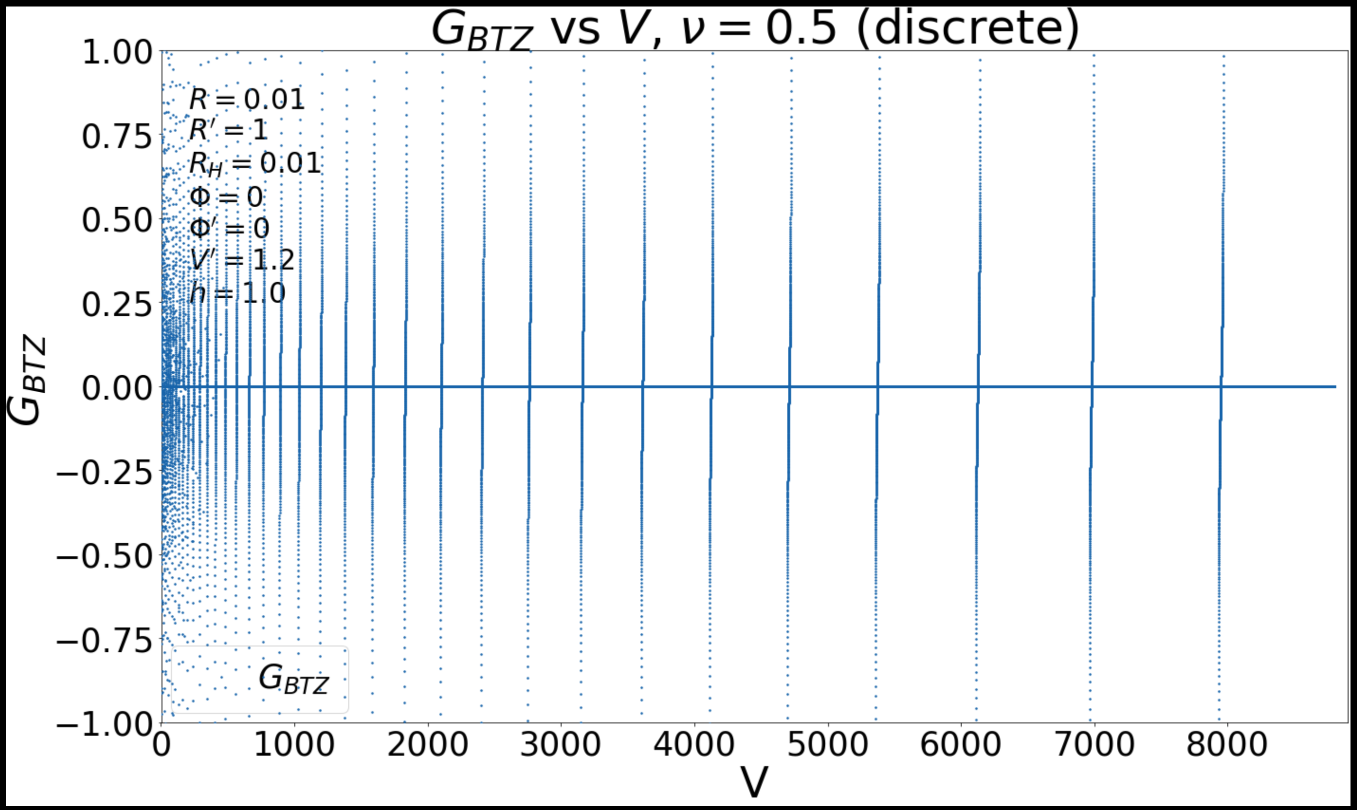

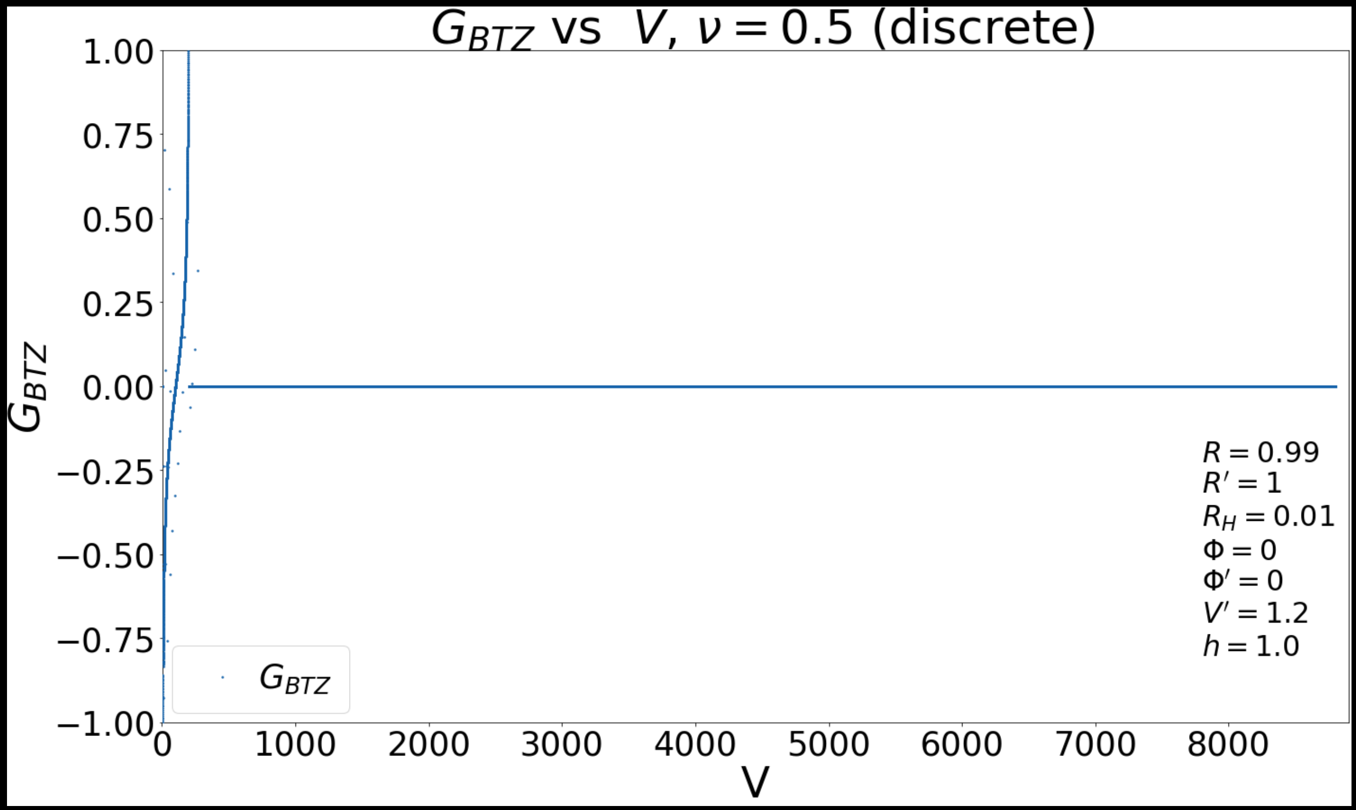

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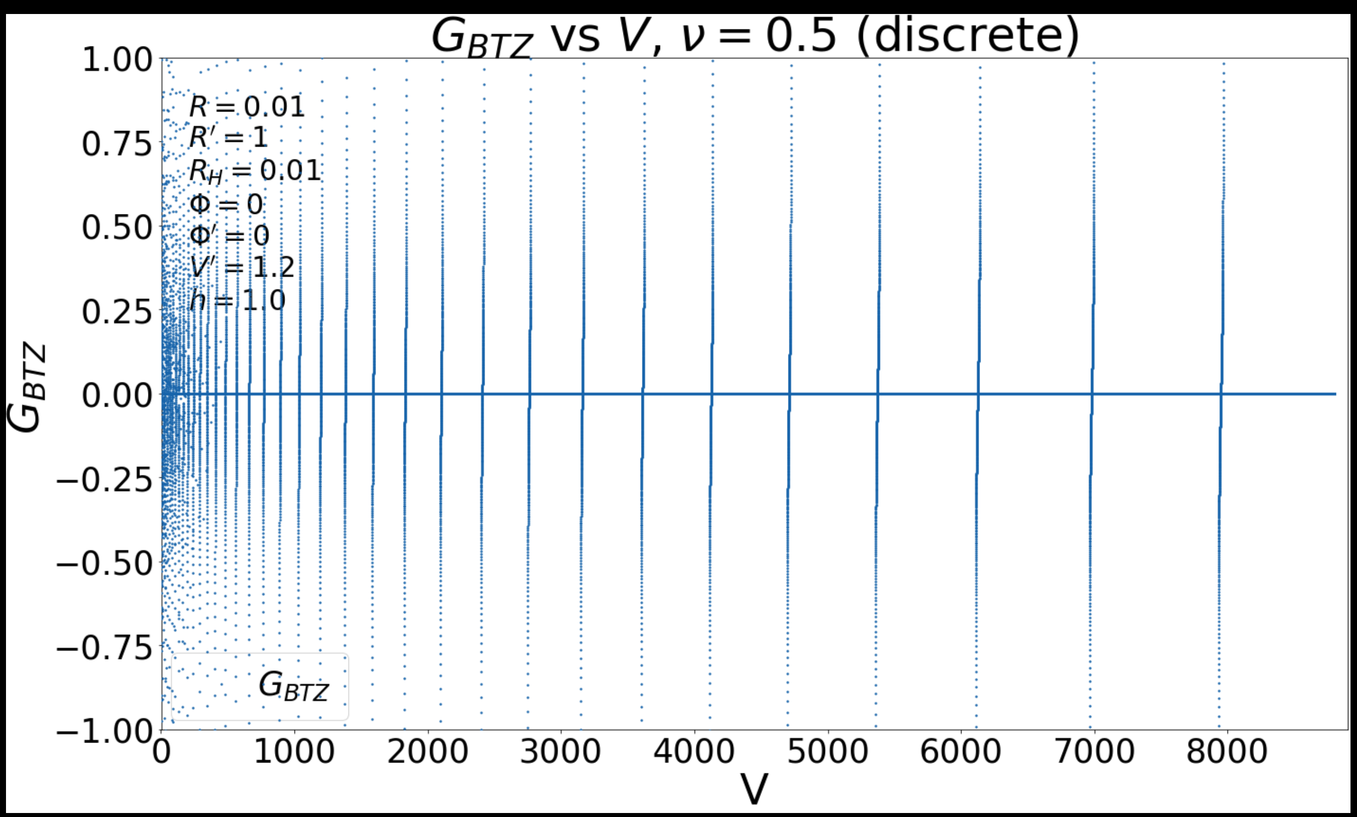

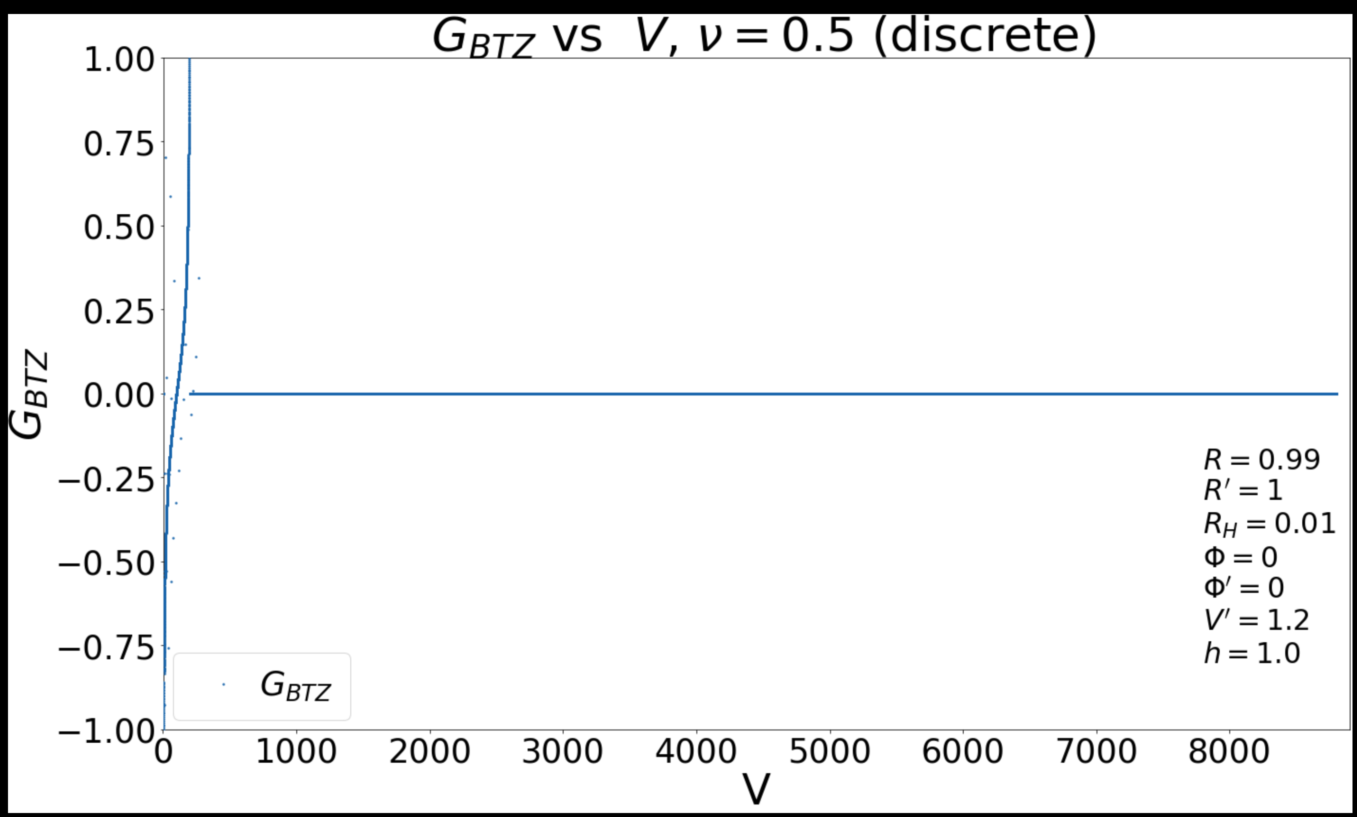

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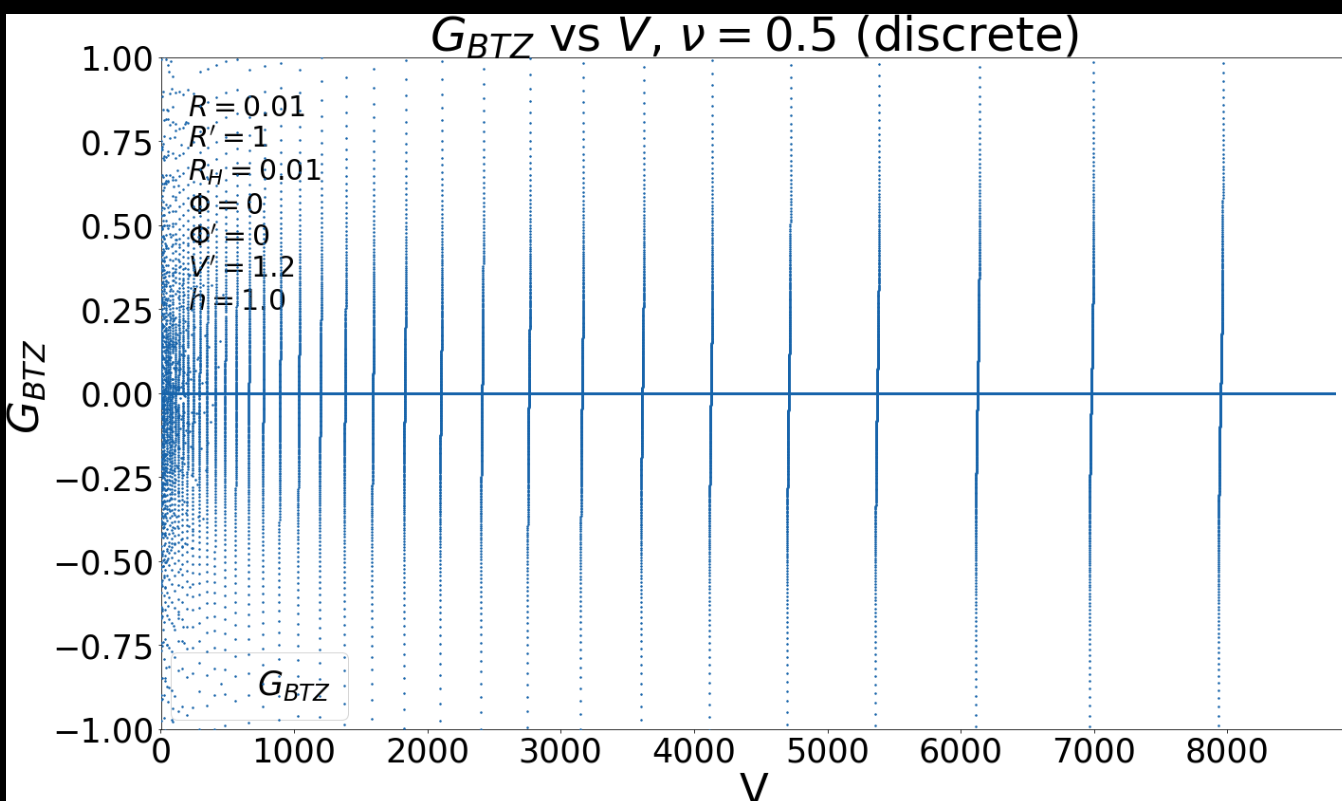

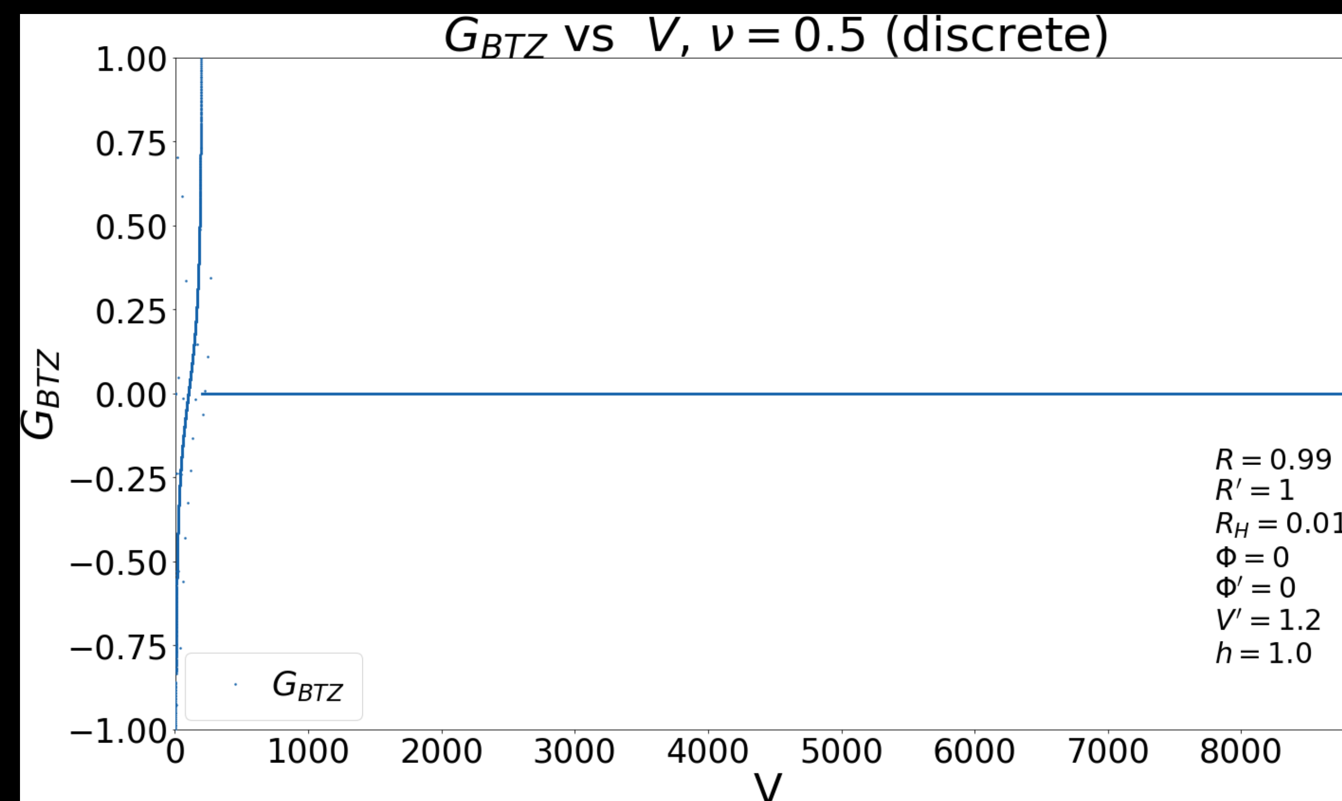

Not clear how to obtain a decay rate for arbitrary initial data

Supplementary

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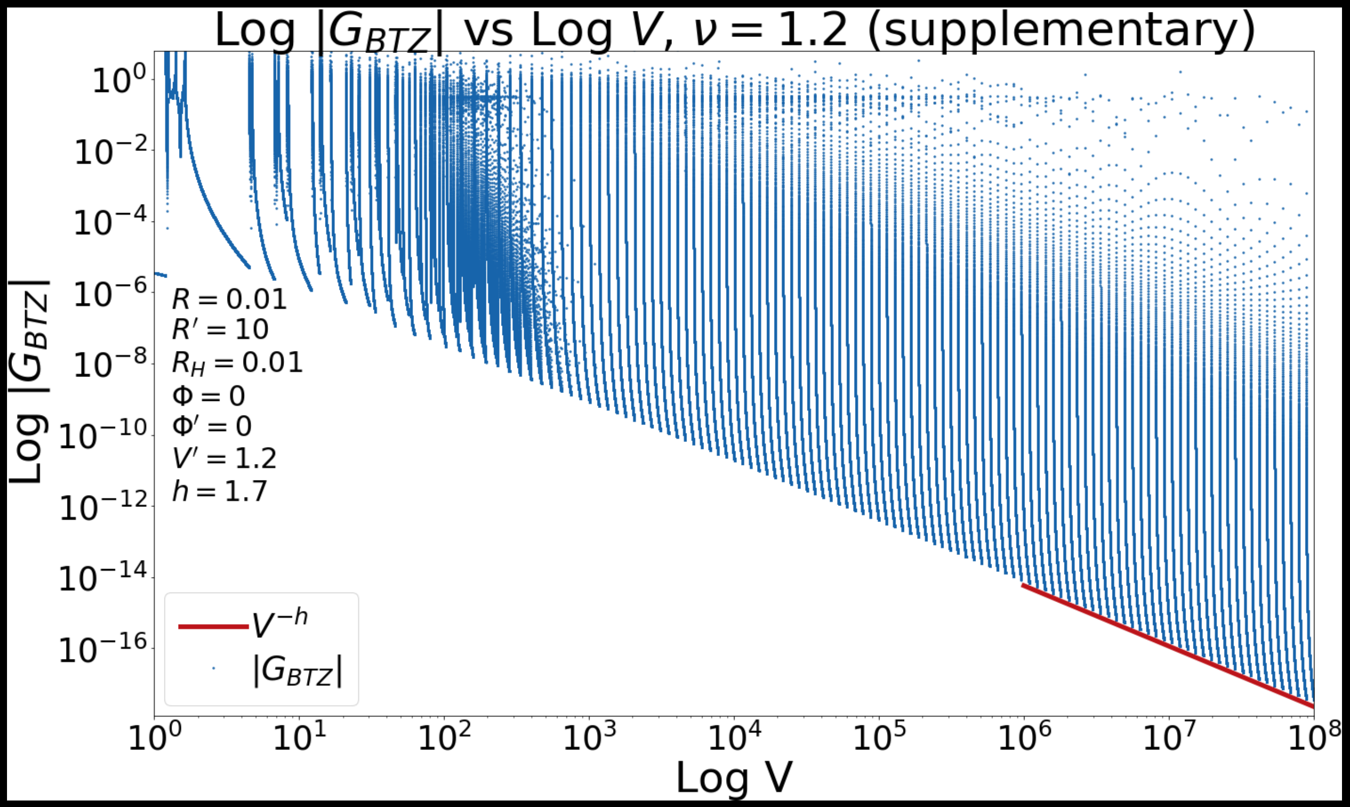

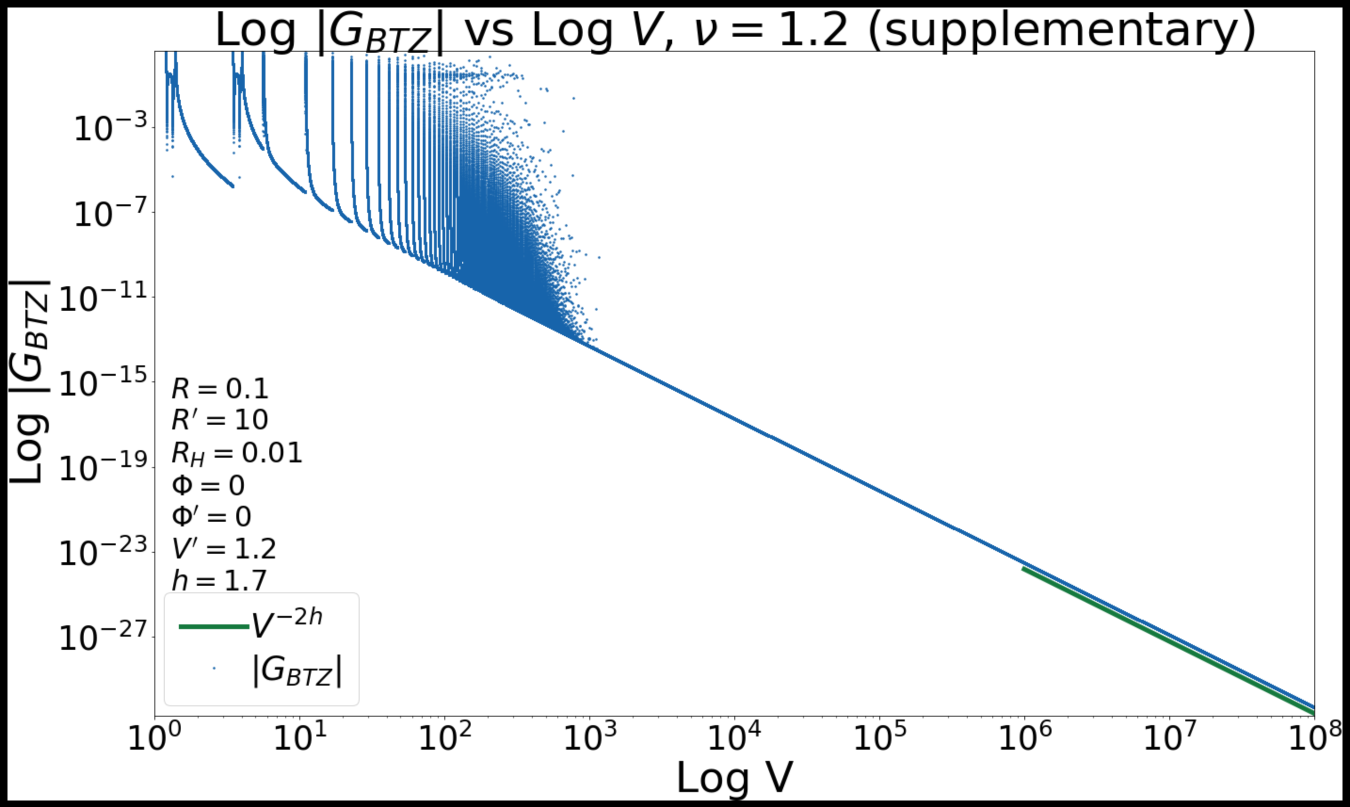

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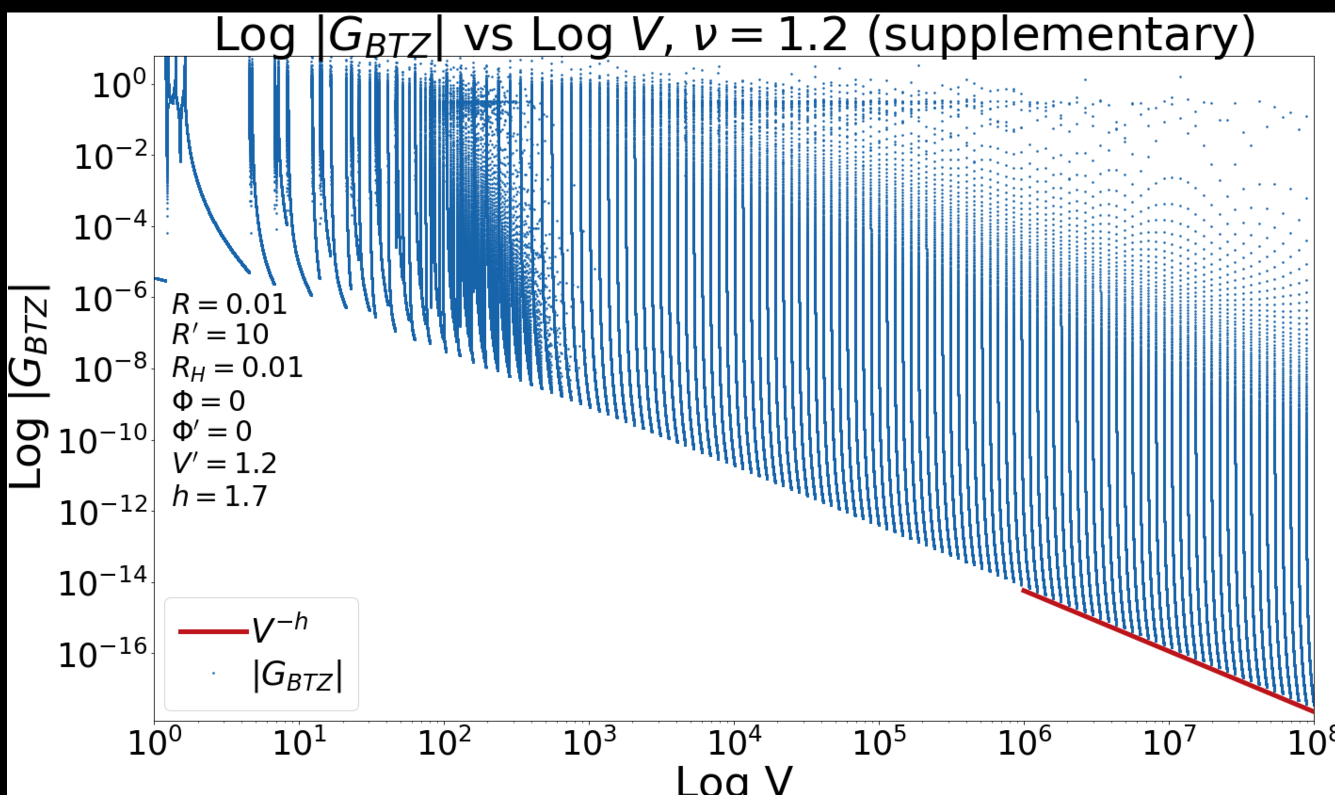

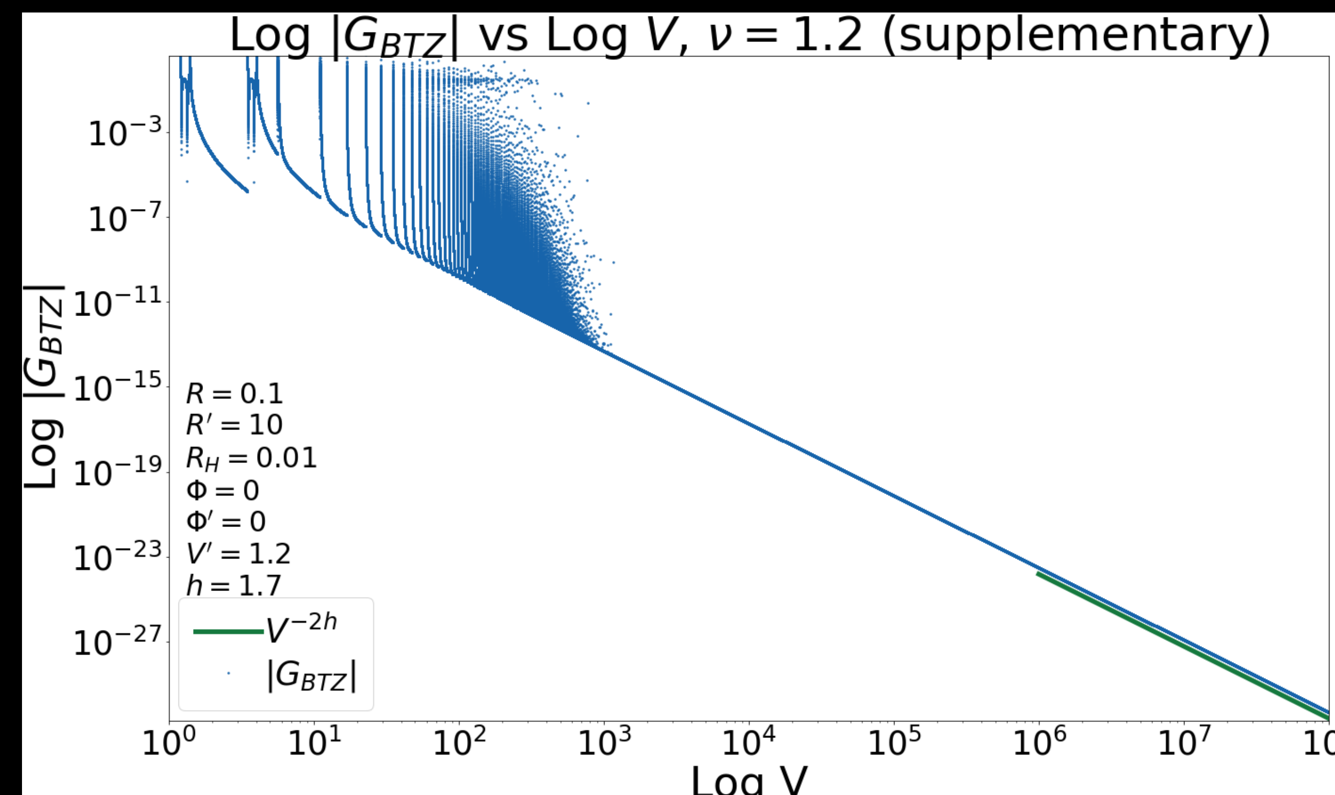

$$h = \frac{1}{2} + \nu$$

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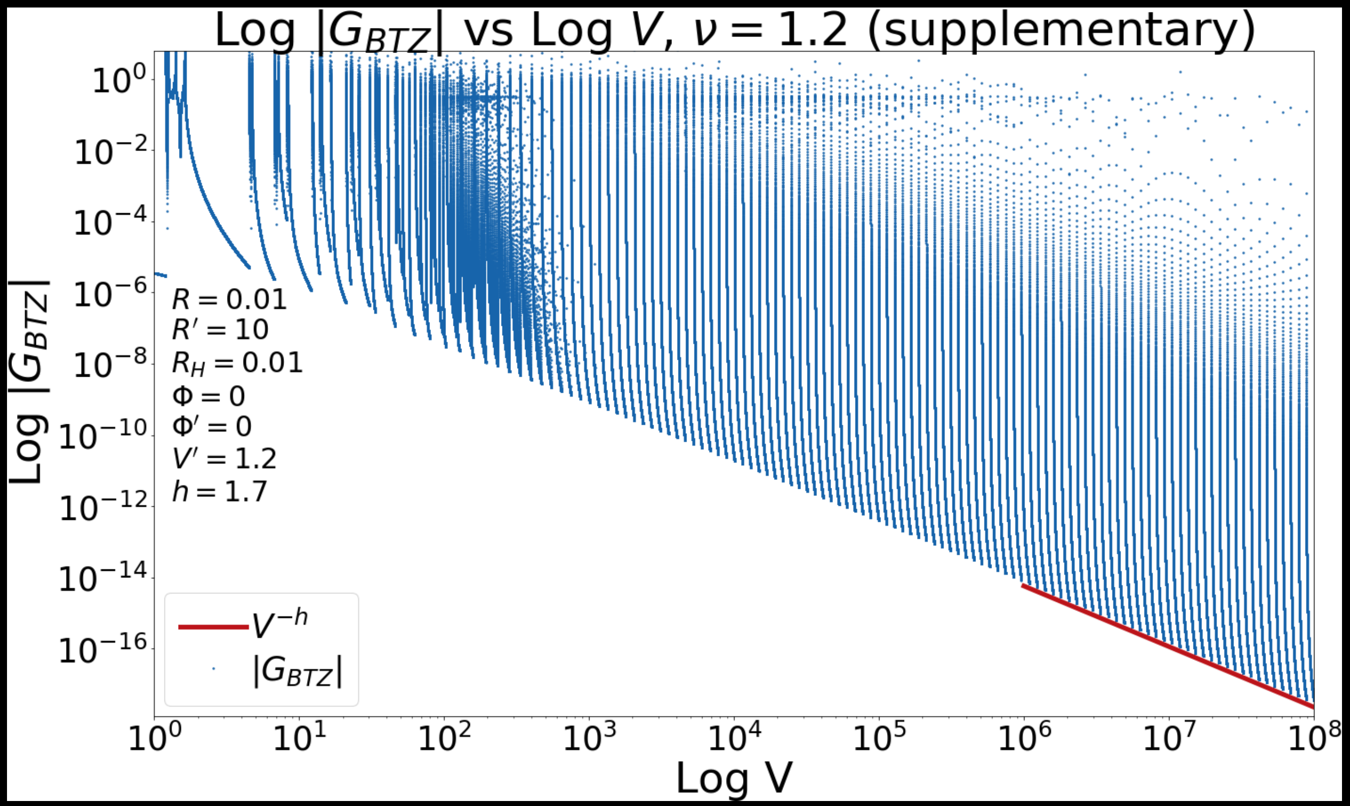
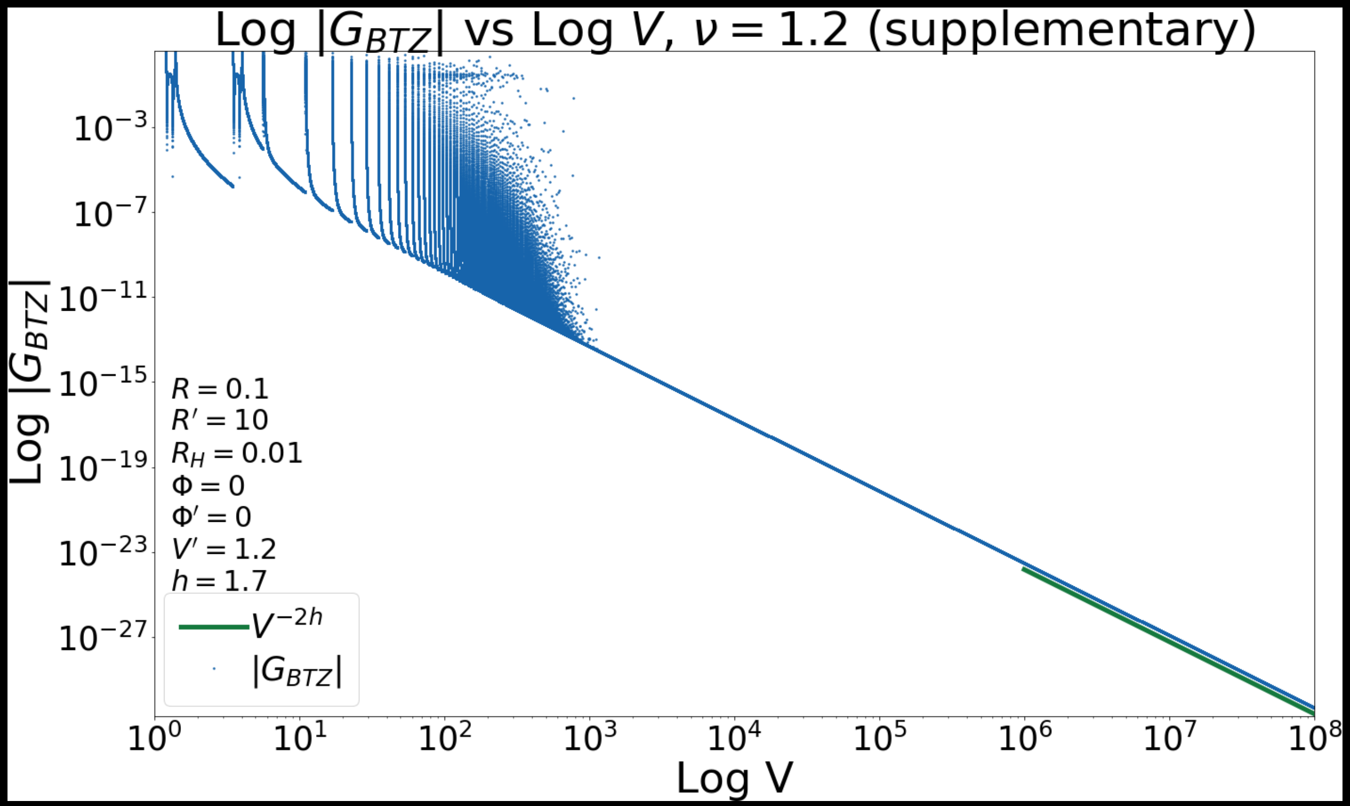
$$G^{\mathrm{AdS}_3}_{\mathrm{ret}} = \frac{\Theta(t-t')\Theta(\Sigma)}{\pi\sqrt{|\Sigma(\Sigma-2)|}} \begin{cases} -\cos(2\nu\arccos(1-\Sigma)) & \text{if} \quad \Sigma < 2 \\ \sin(2\pi\nu)e^{-2\nu\,\mathrm{arccosh}(\Sigma-1)} & \text{if} \quad \Sigma > 2 \end{cases}$$





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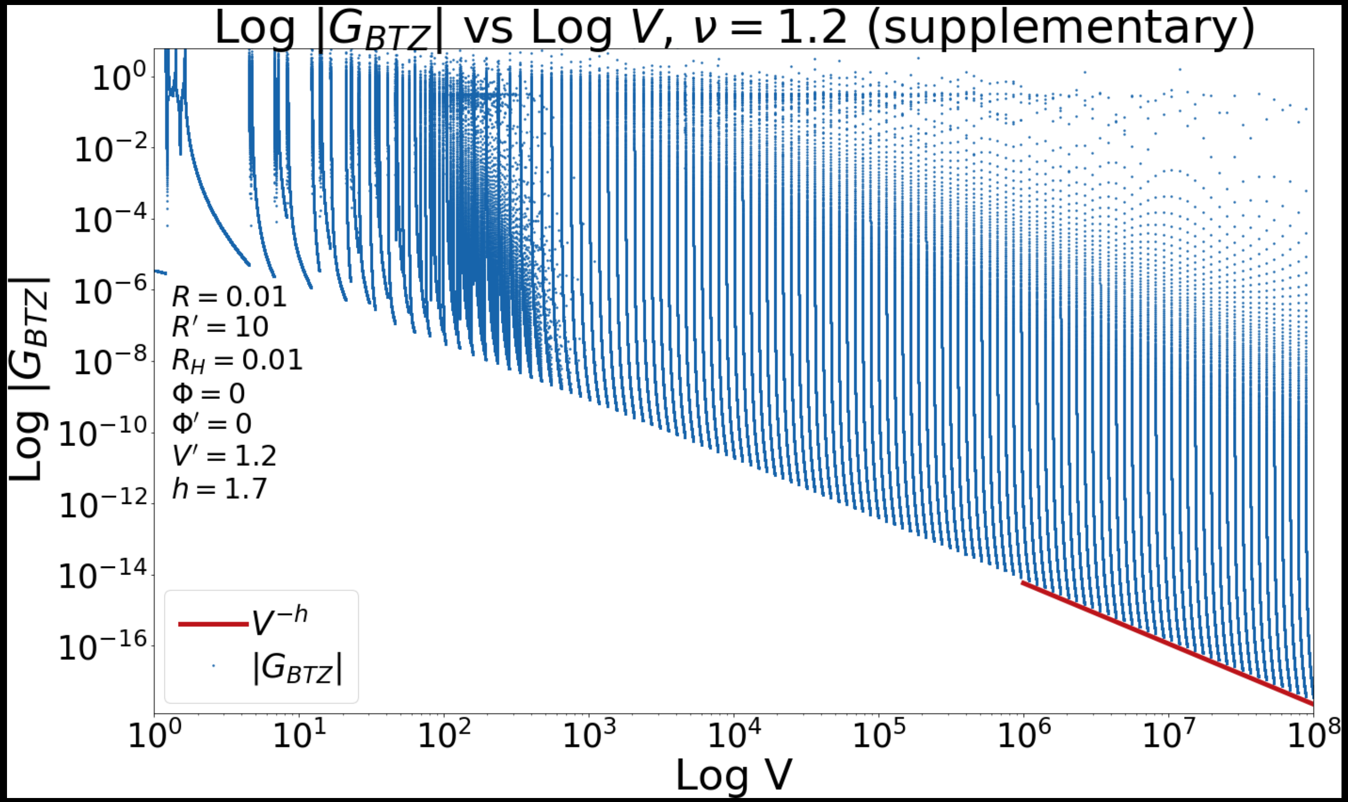

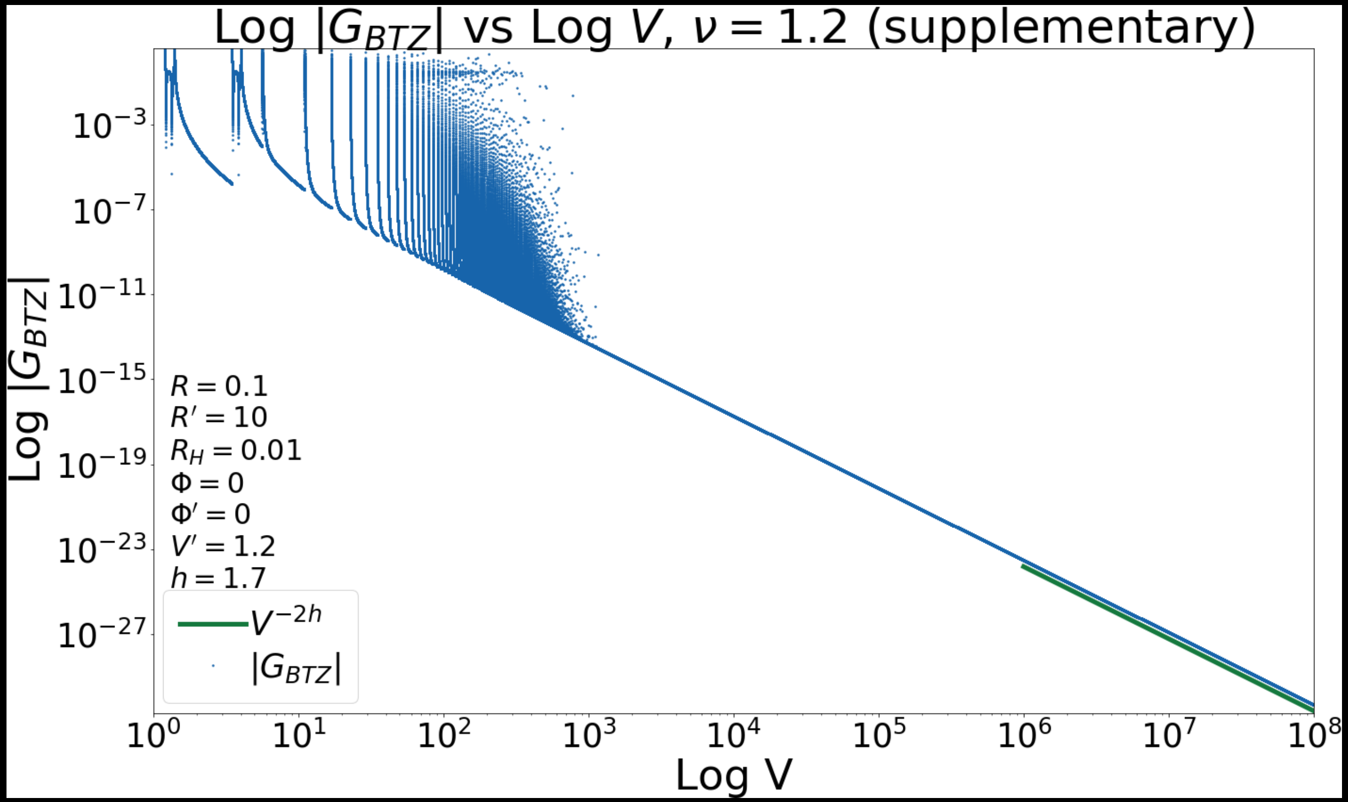

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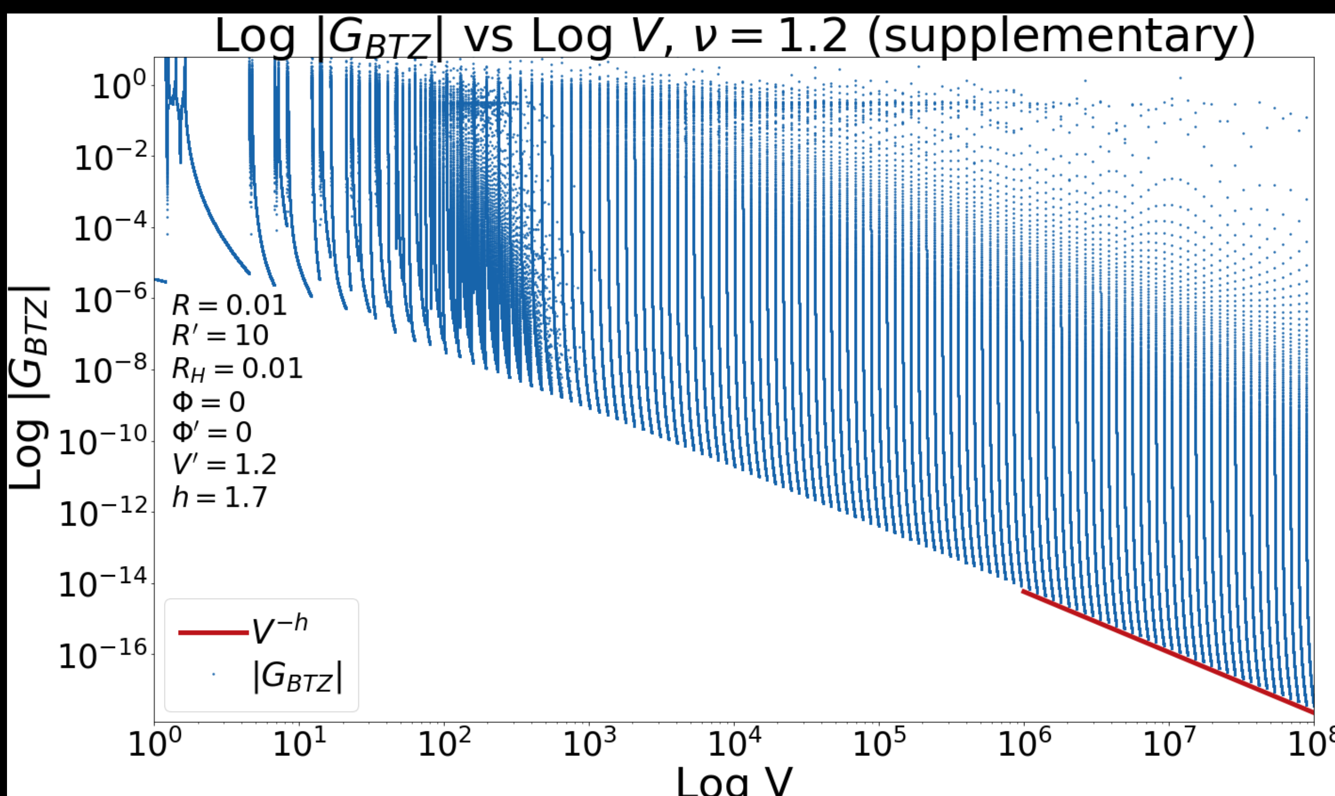
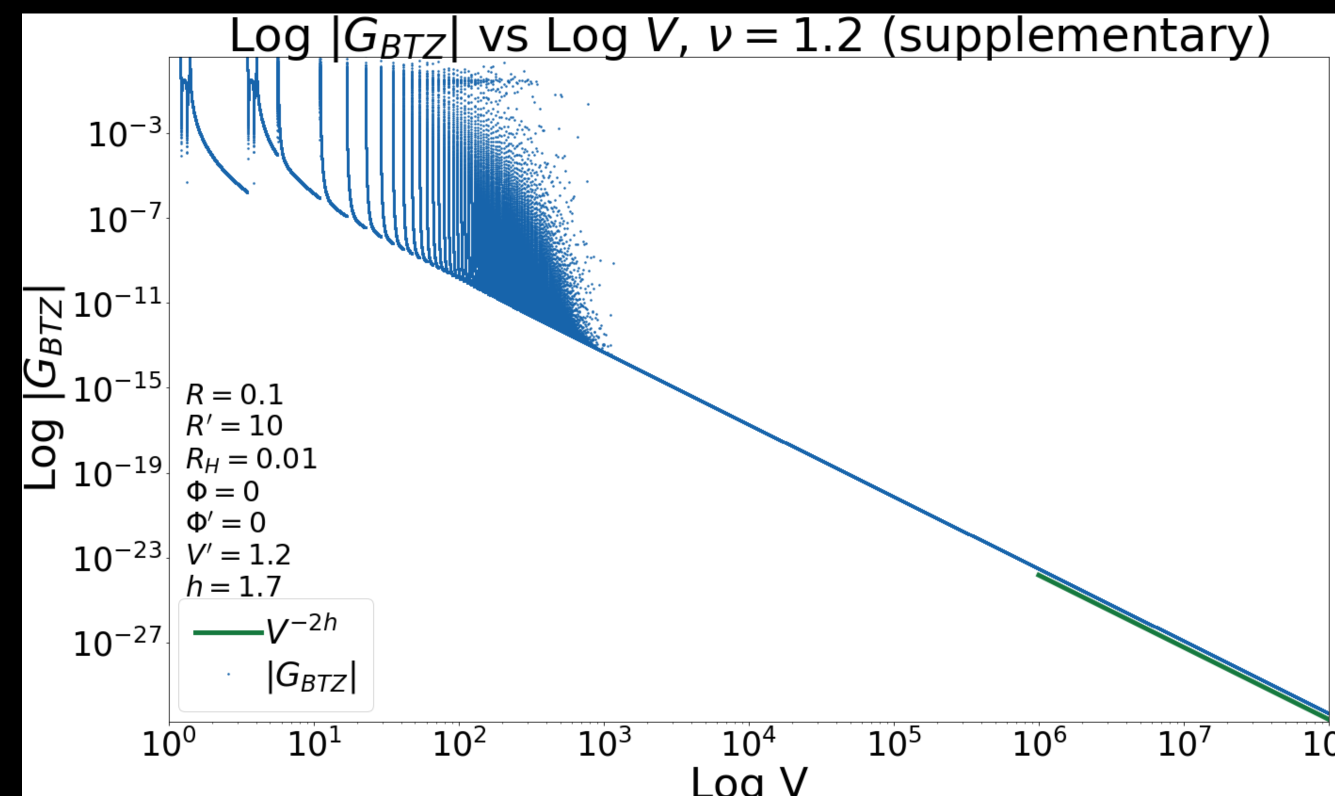
Spikes -> Null geodesics





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Null Geodesics

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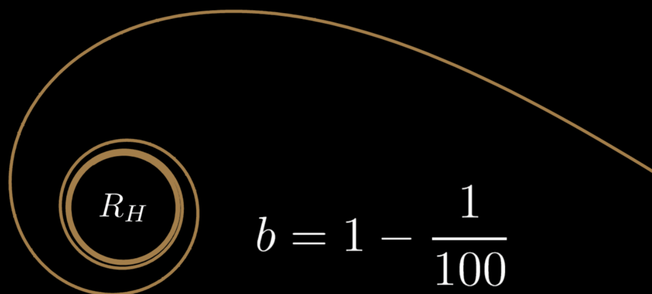
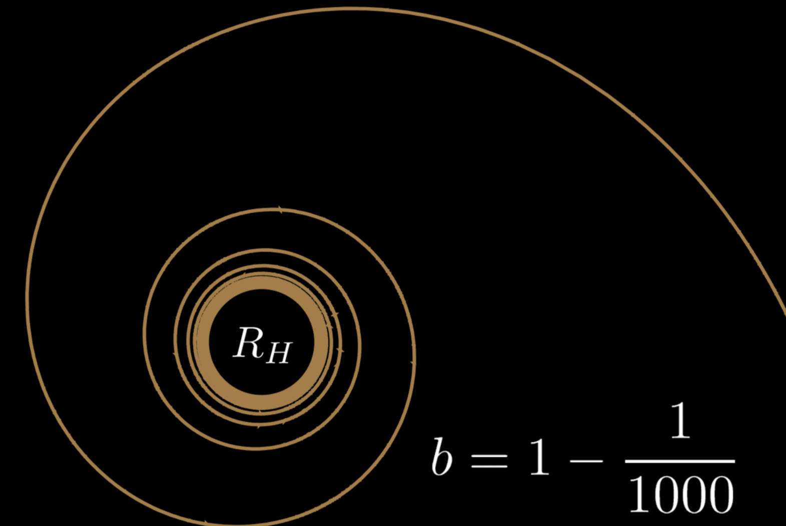


$$b = 1 - \frac{1}{100}$$



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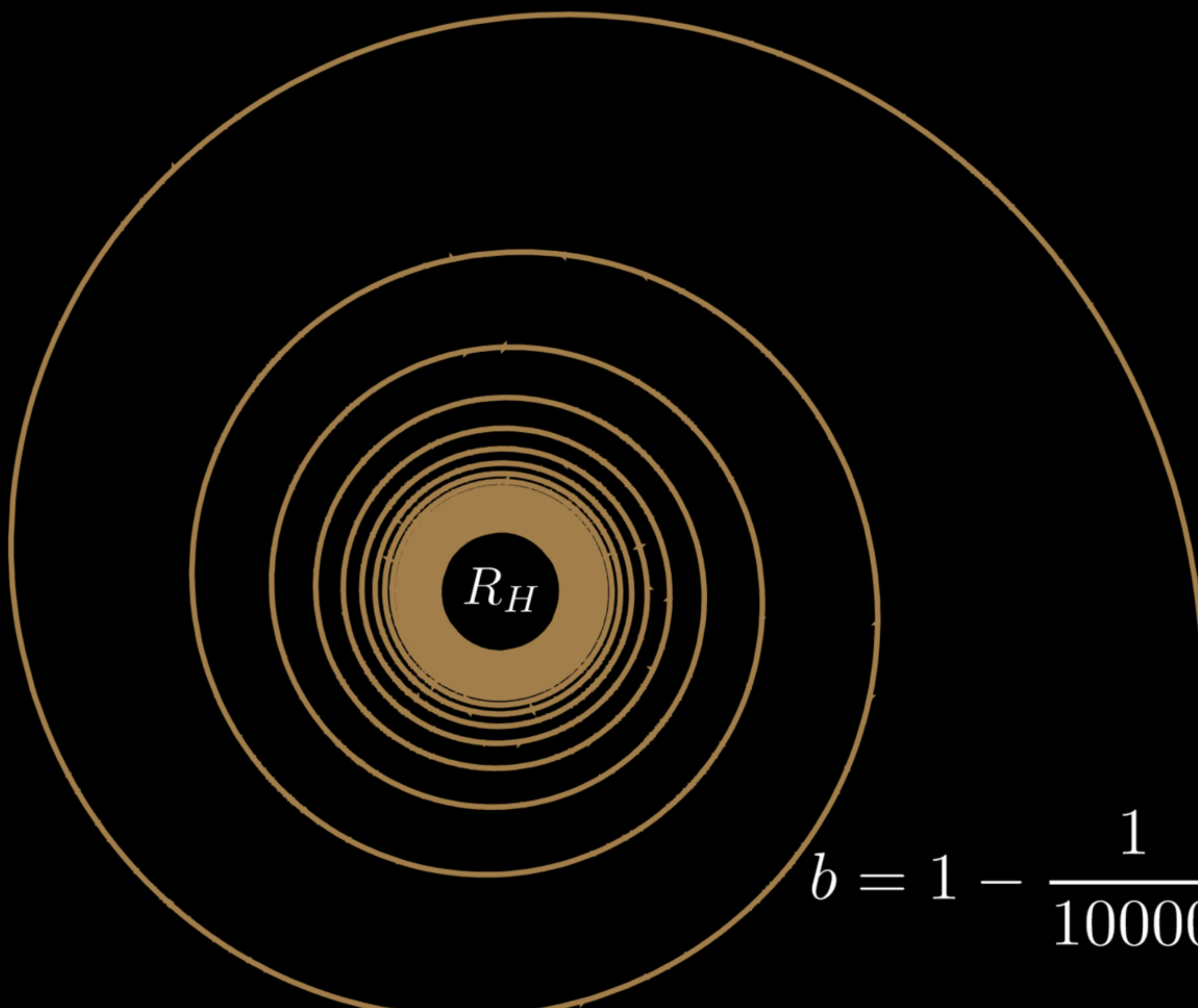

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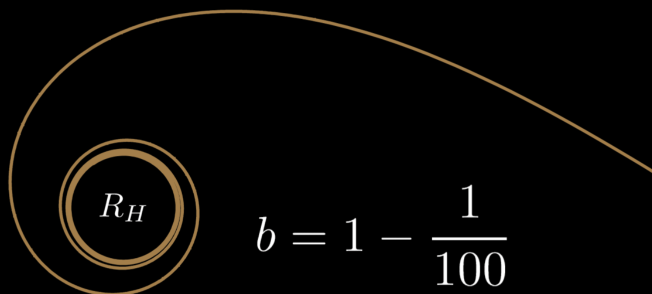
Null Geodesics



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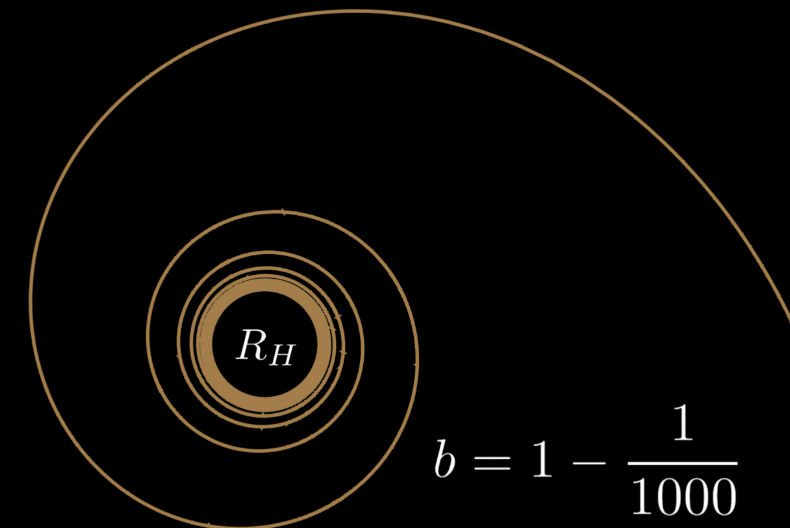

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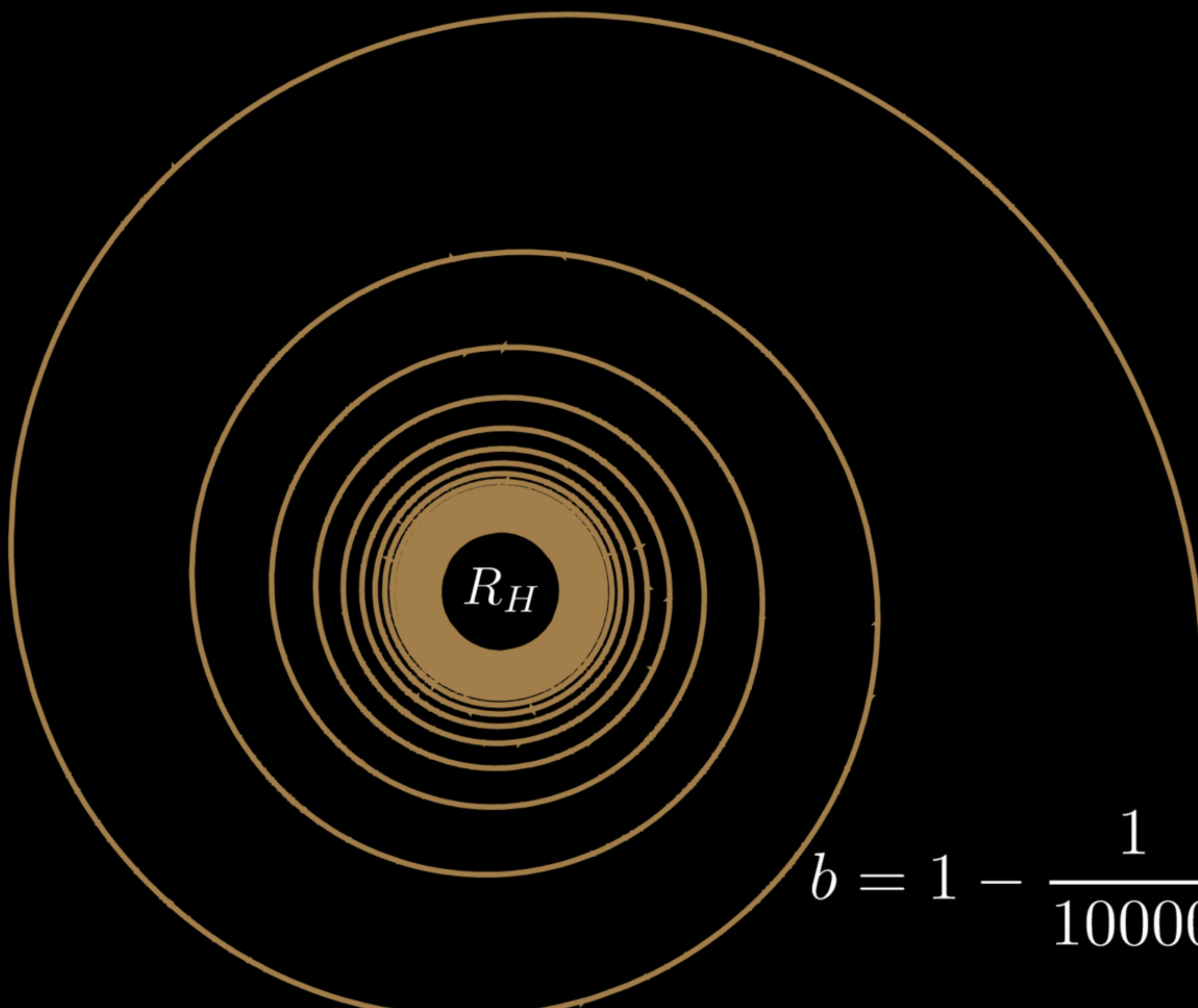

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Geodesics parameterised by $b = L/E$

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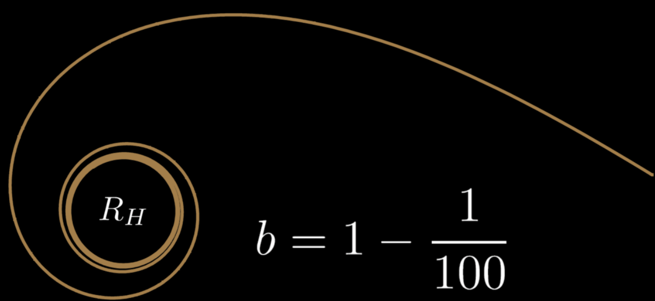

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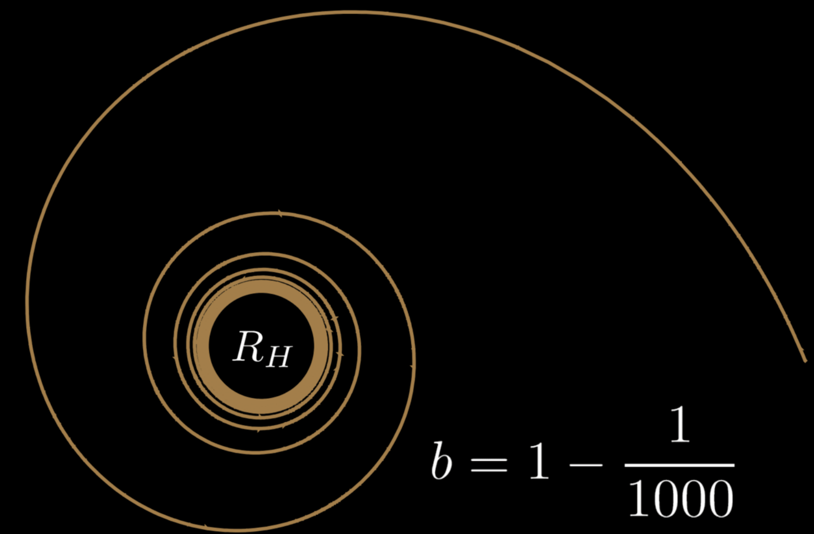

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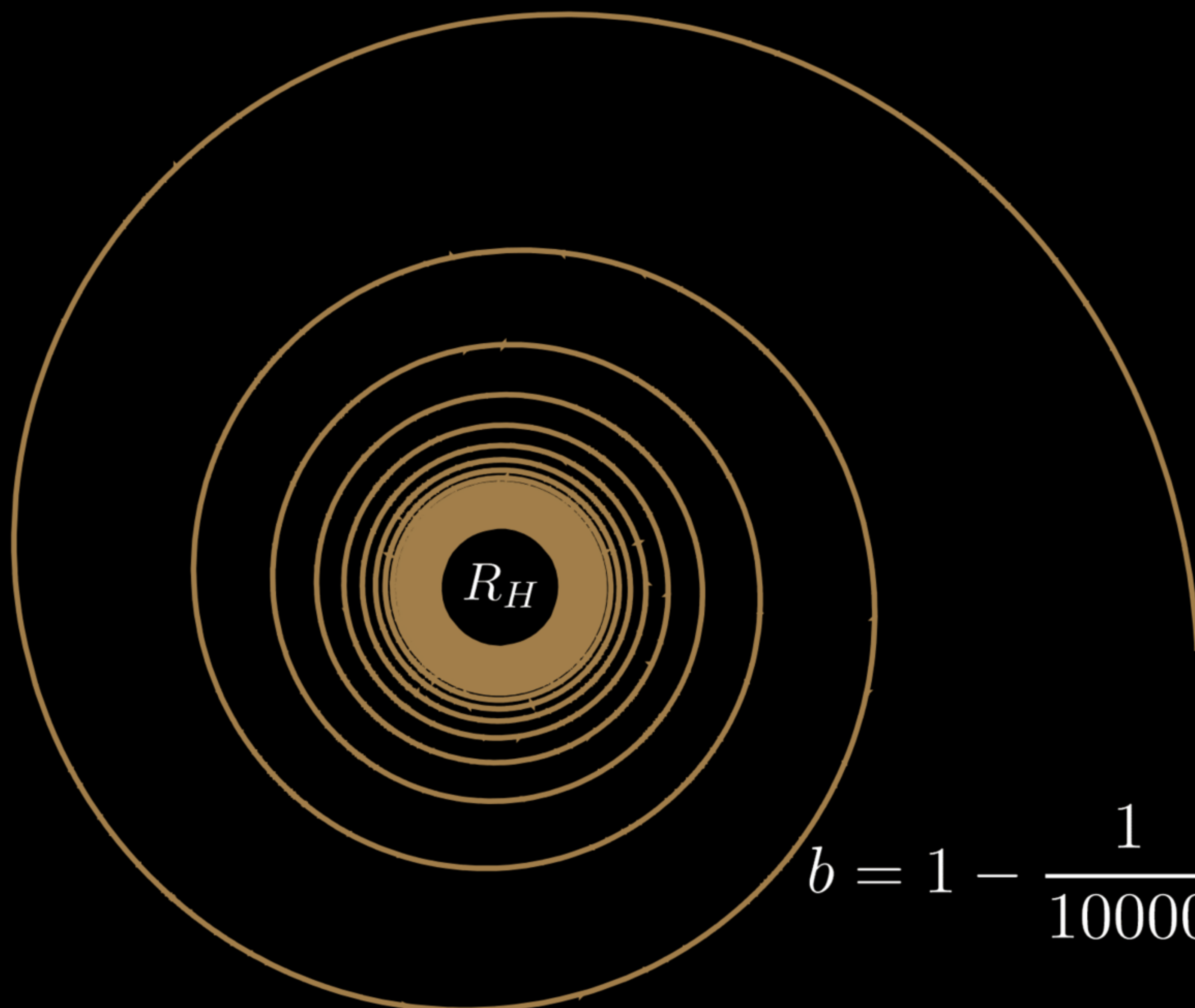

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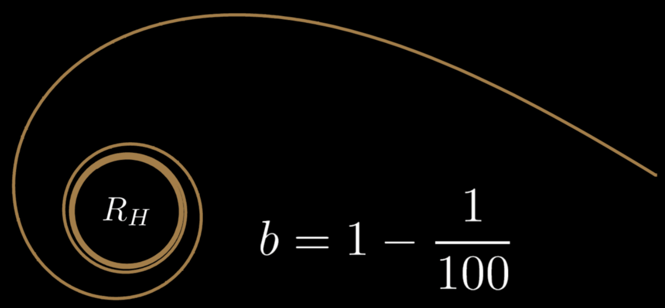
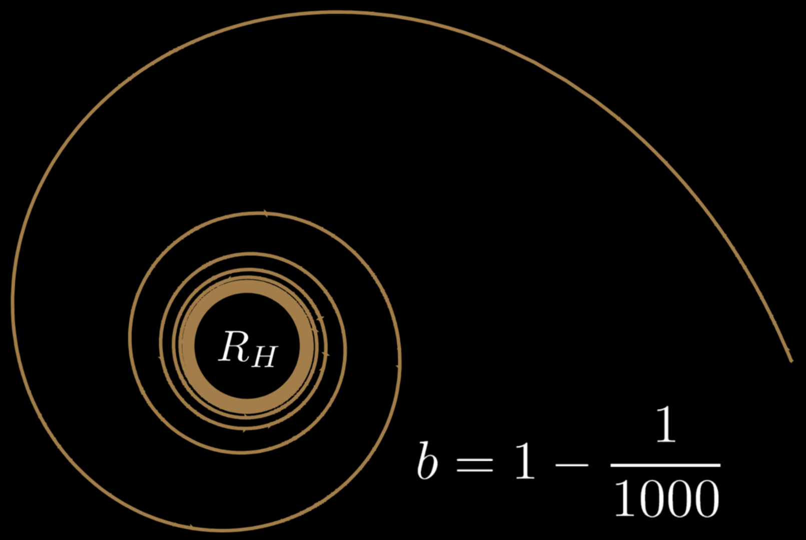
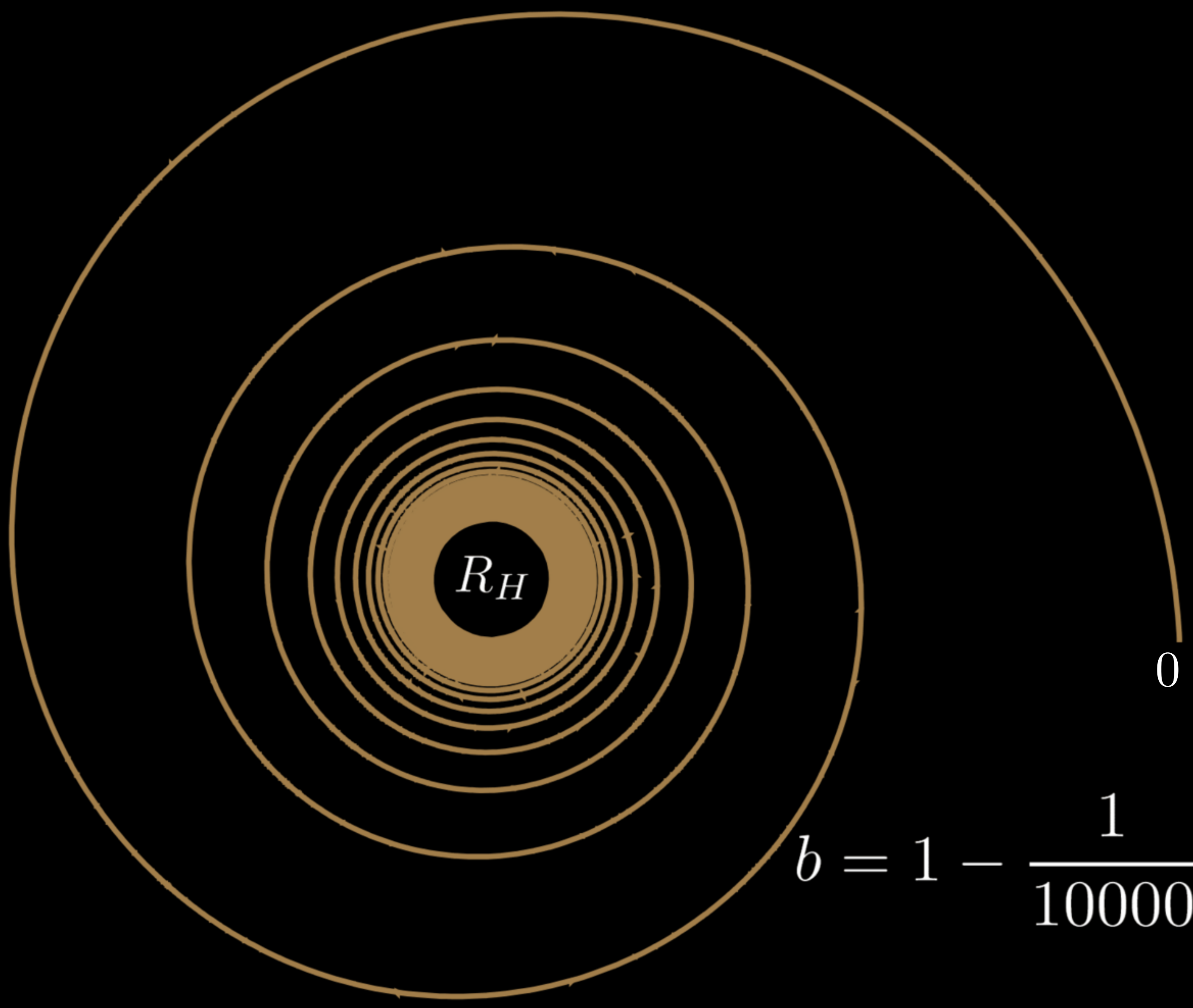
$b = 1$ geodesics are trapped on the horizon

Null Geodesics







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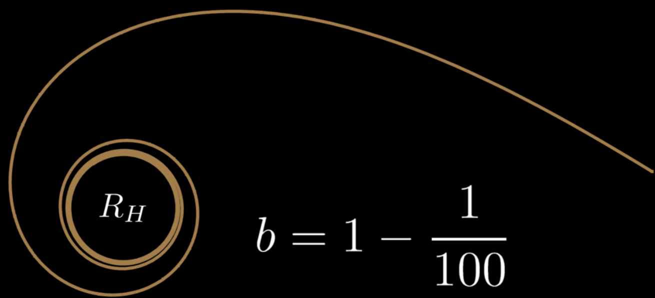

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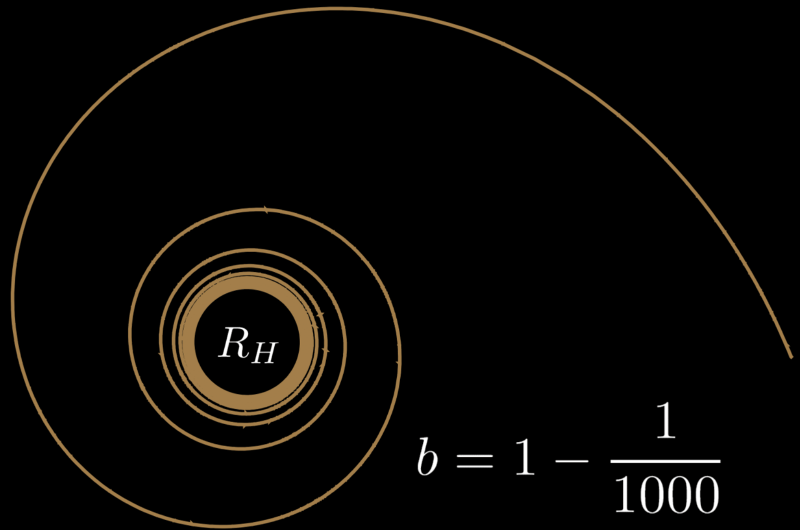

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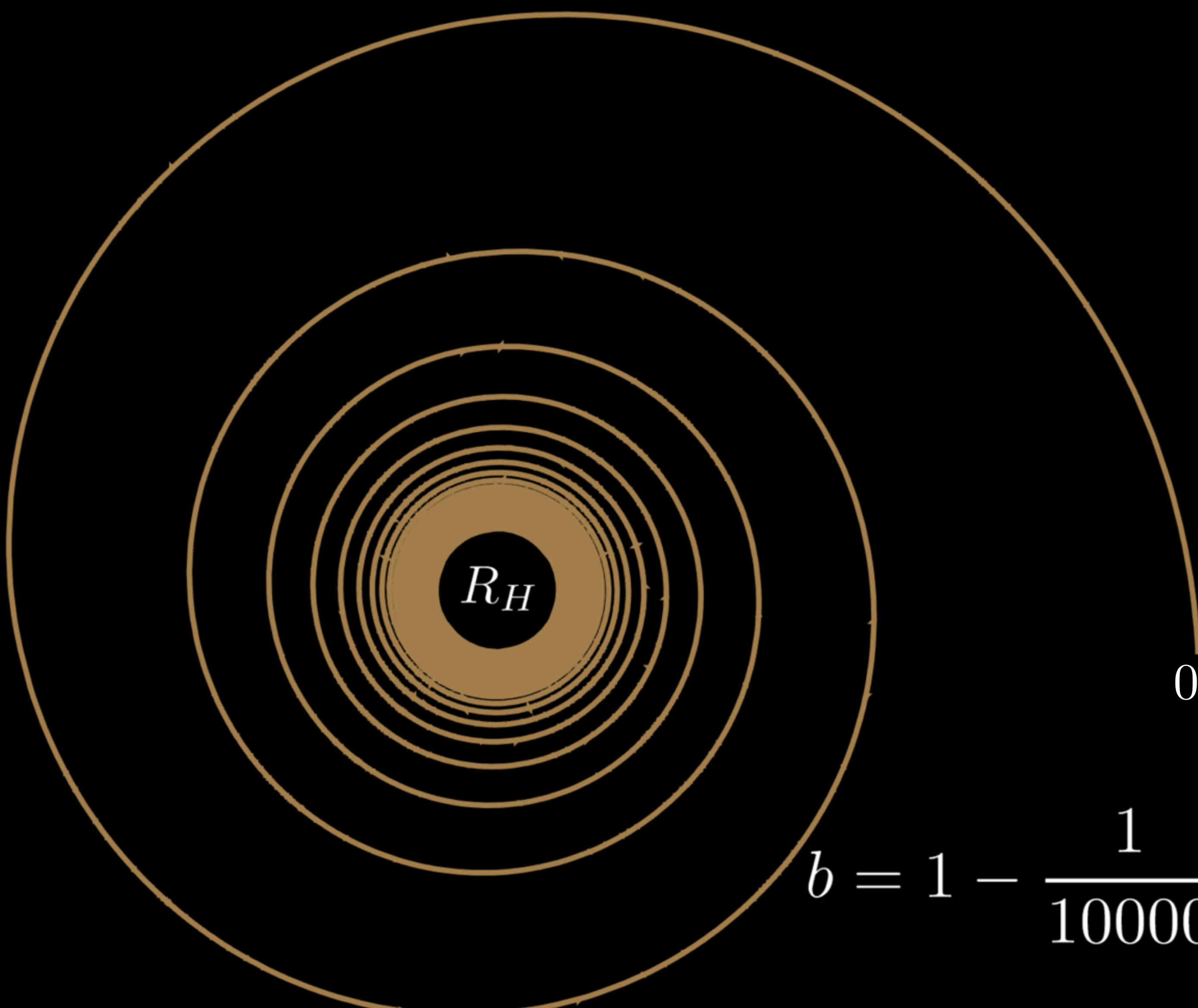

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Cause the instability on the horizon!

Initial data extending to horizon

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Putting the field and source on the horizon doesn’t give any decay rates

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Field - on, Source - near => Intermediate times will give us a hint as to what to expect in the case where both field and source points are on the horizon

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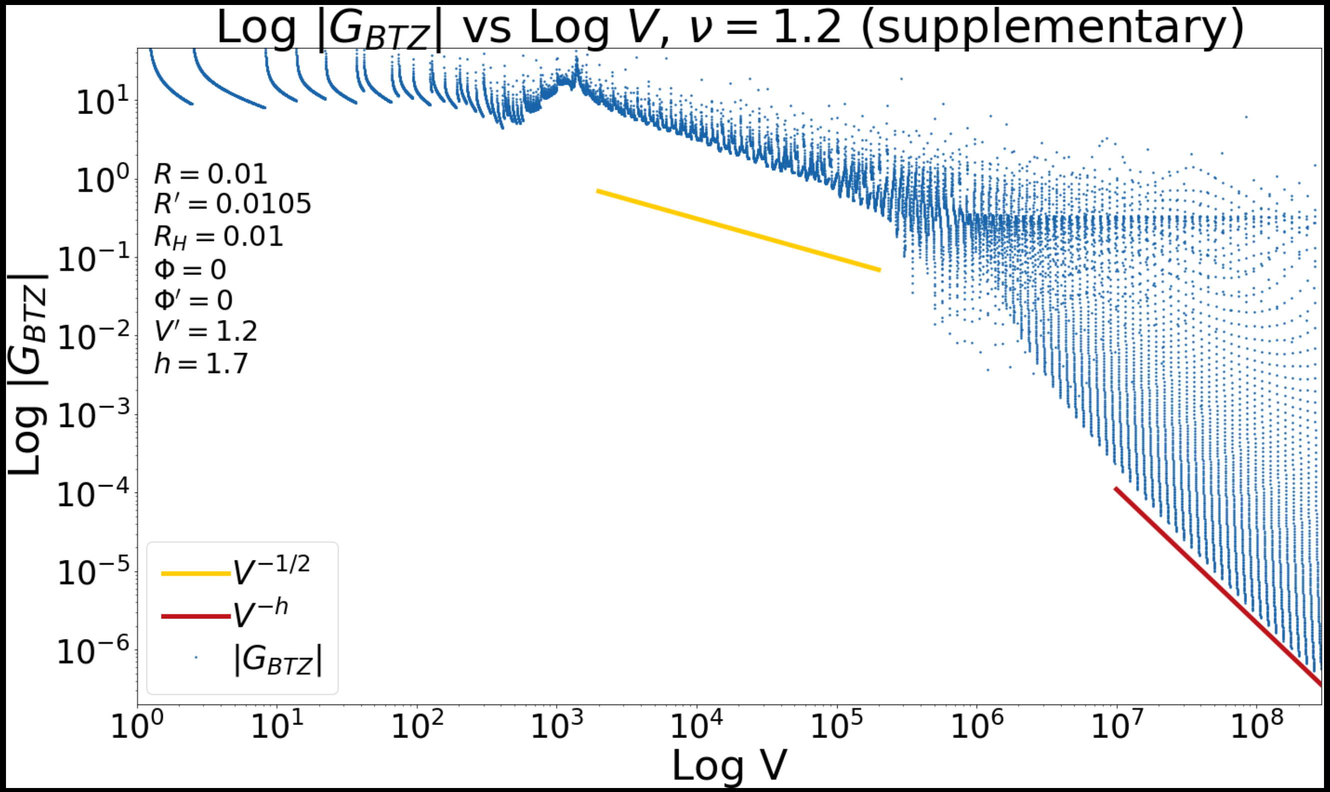

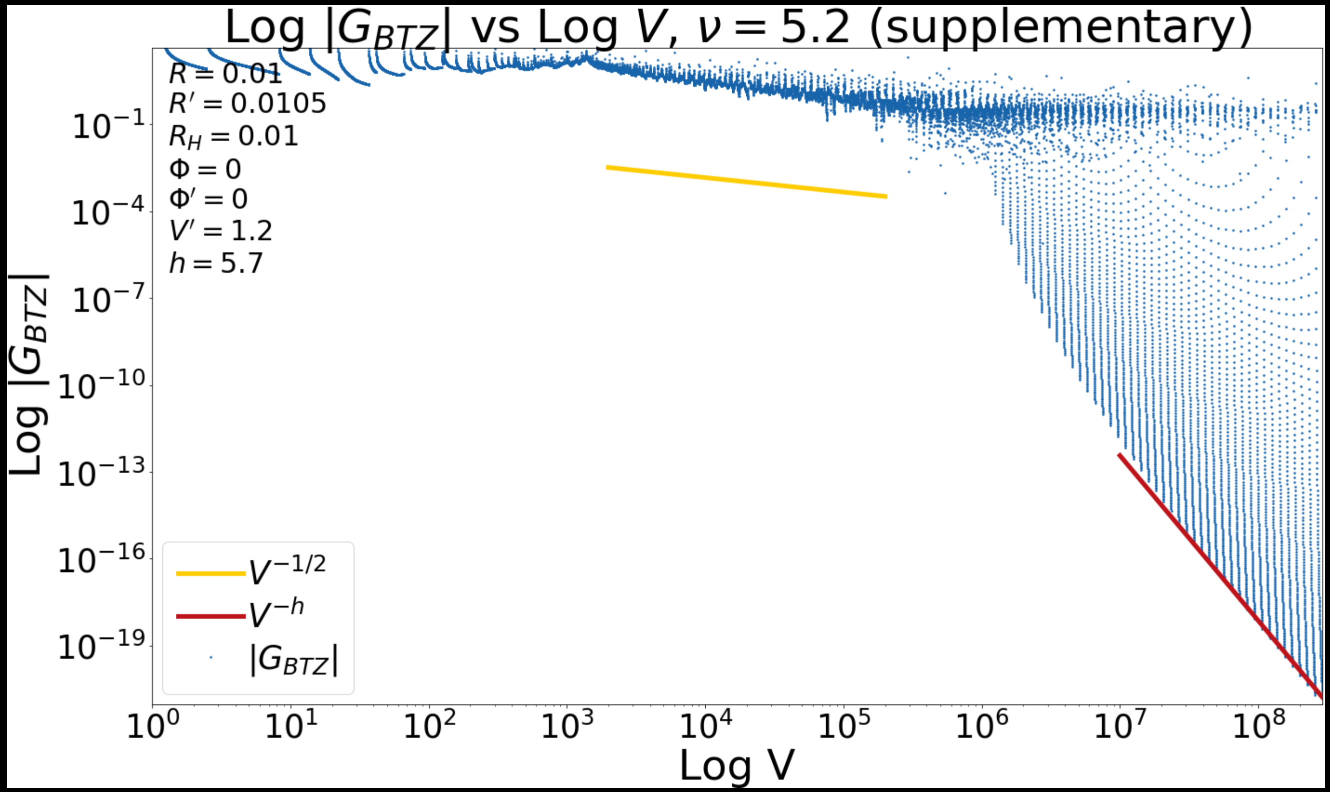

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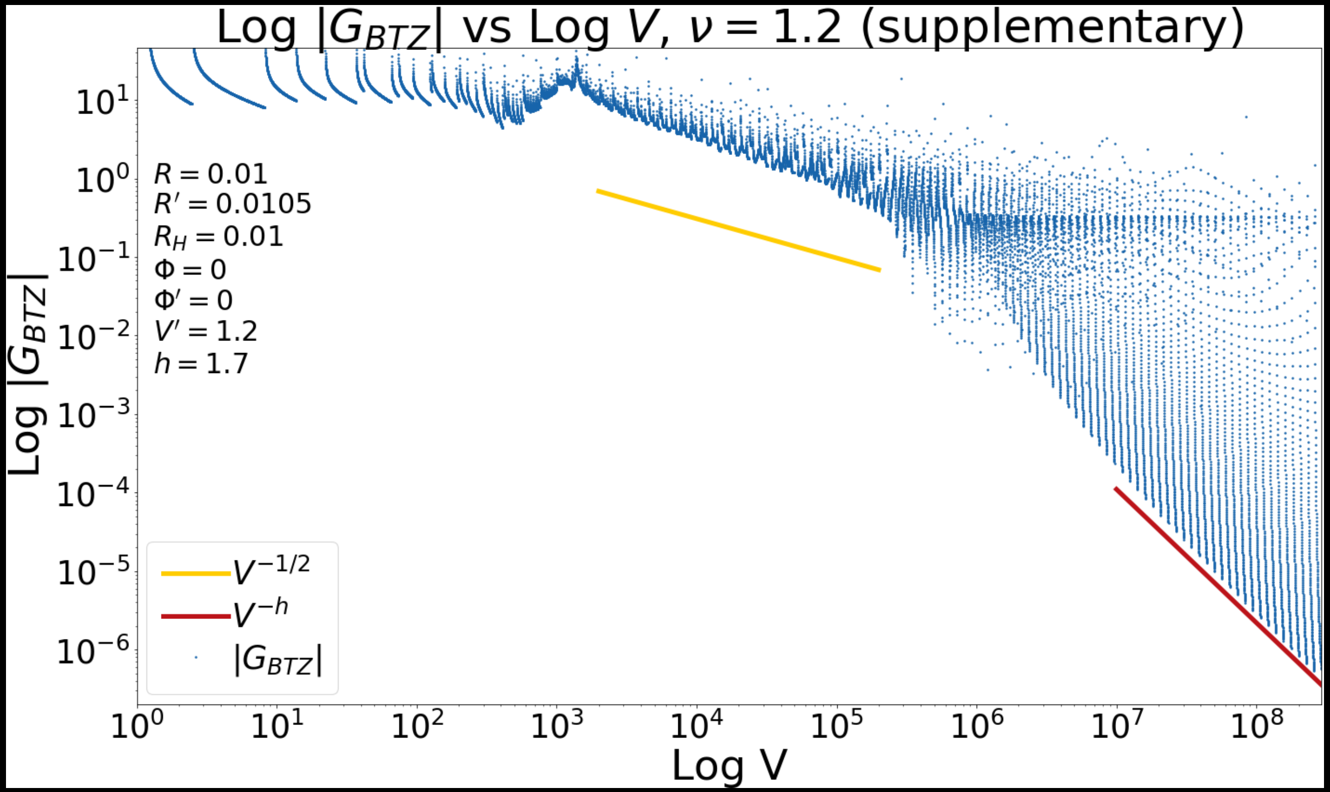
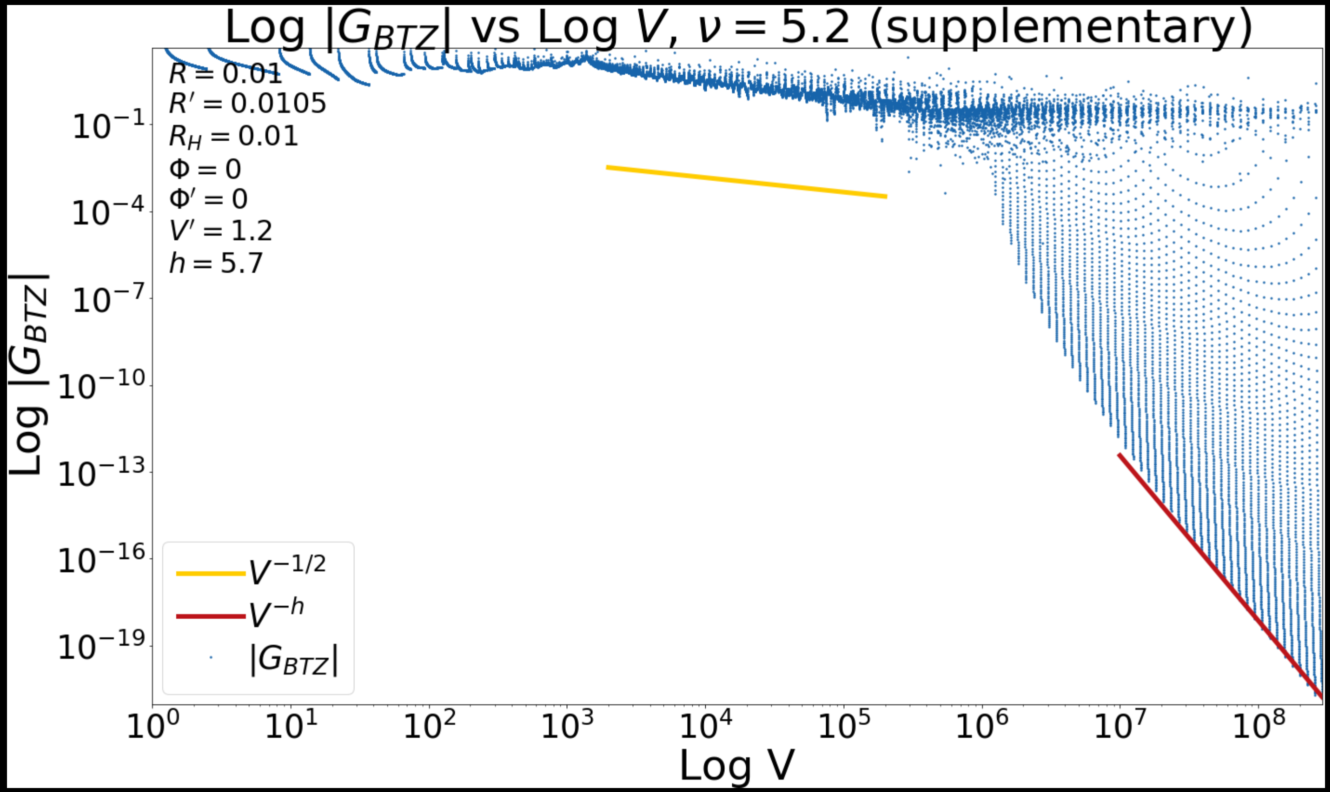
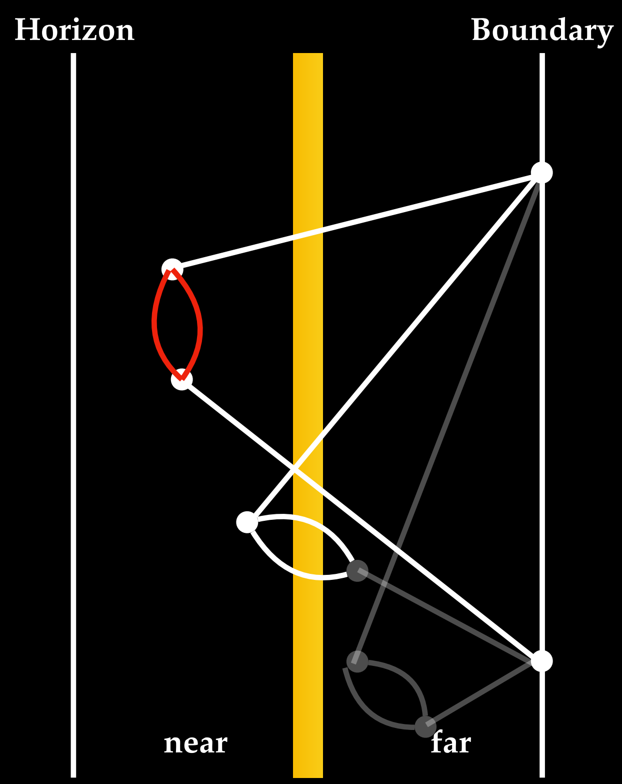
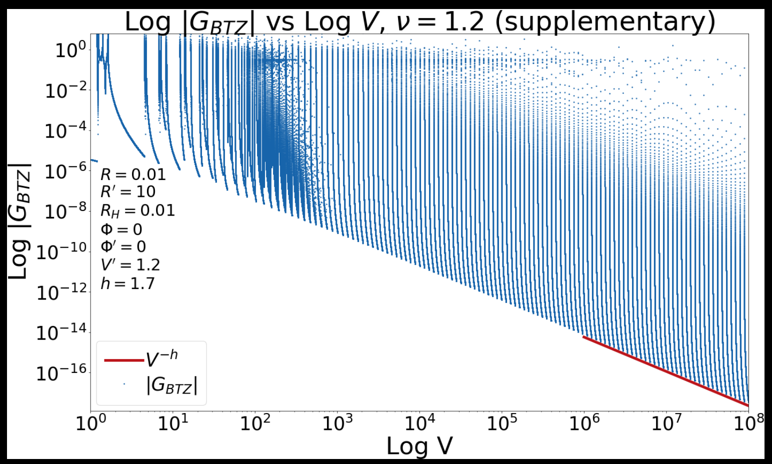


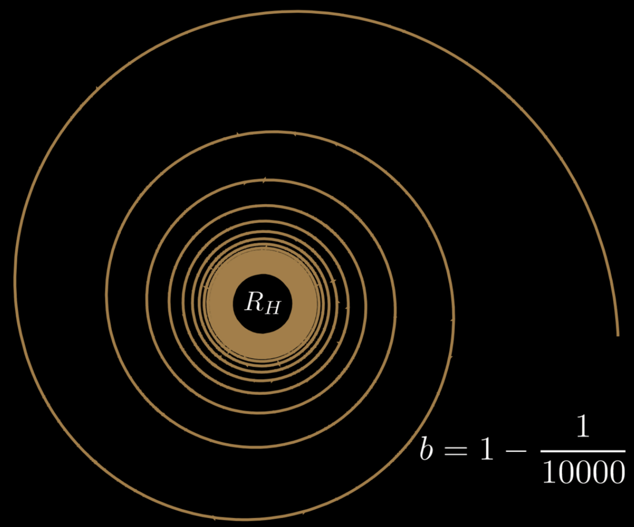


Intermediate slope independent of mass of perturbing field!

Outlook

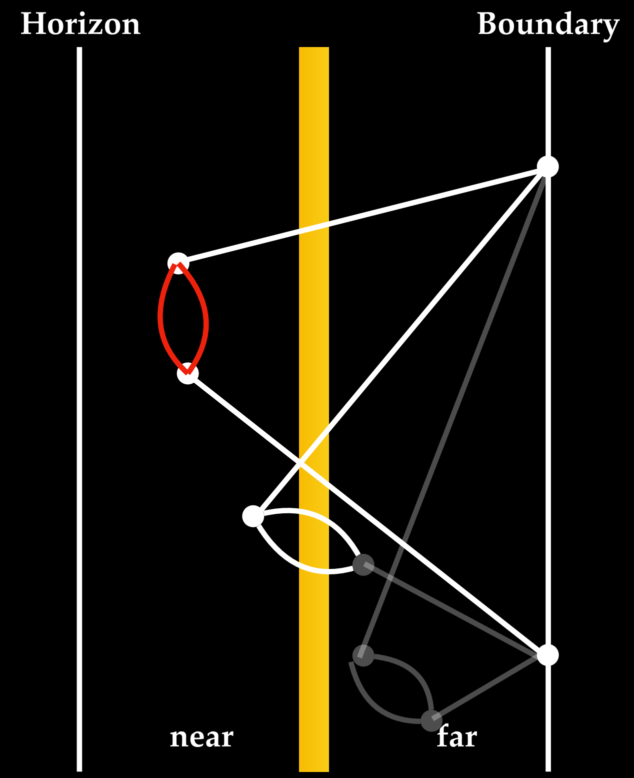

Horizon
Boundary
near
far

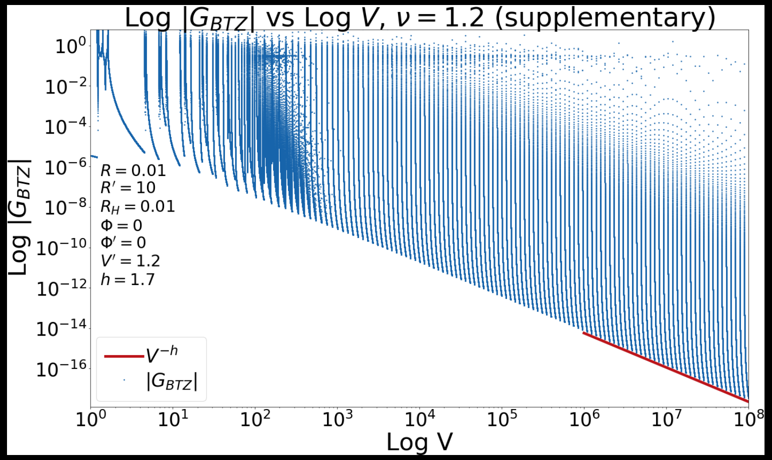

Log $|G_{BTZ}|$ vs Log V, $\nu = 1.2$ (supplementary)
Log $|G_{BTZ}|$
Log V
$R = 0.01$
$R' = 10$
$R_H = 0.01$
$\Phi = 0$
$\Phi' = 0$
$V' = 1.2$
$h = 1.7$
V^{-h}
$|G_{BTZ}|$
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^1
10^2
10^3
10^4
10^5
10^6
10^7
10^8

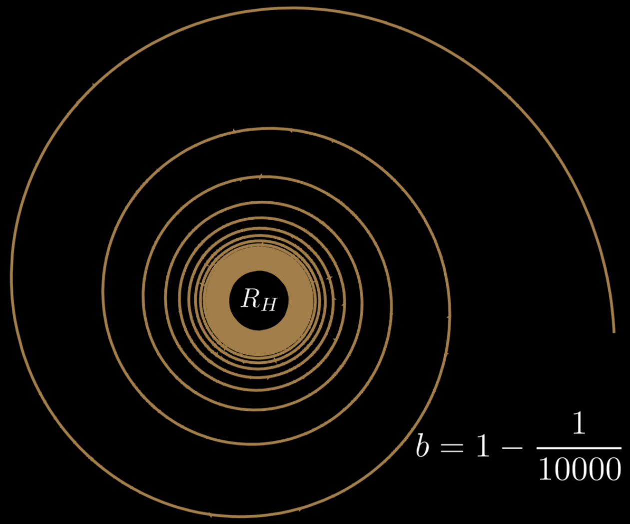

R_H
$b = 1 - \frac{1}{10000}$

Outlook

The Aretakis instability persists in spacetime with non-compact horizon topology







Outlook

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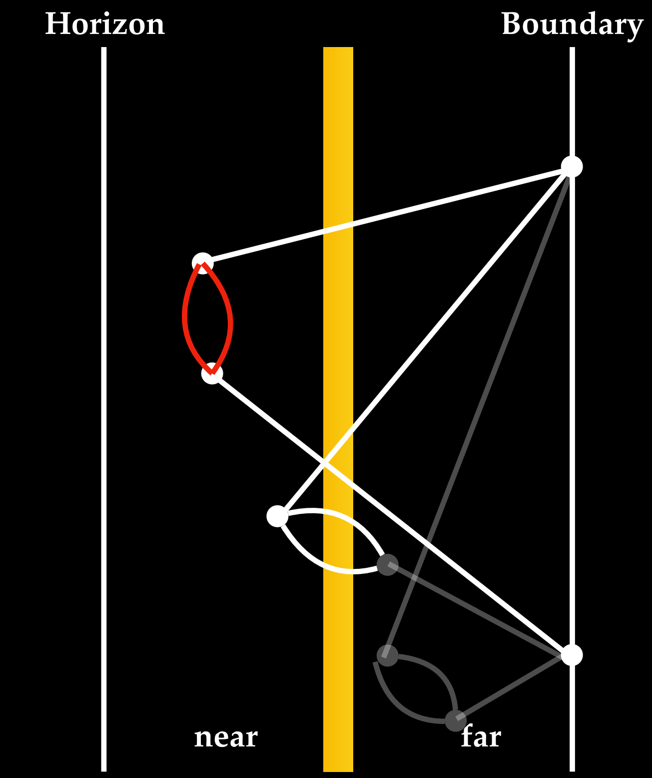
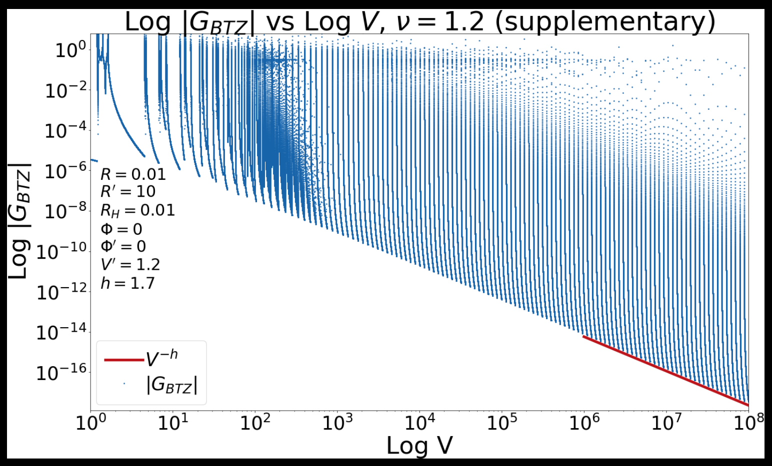
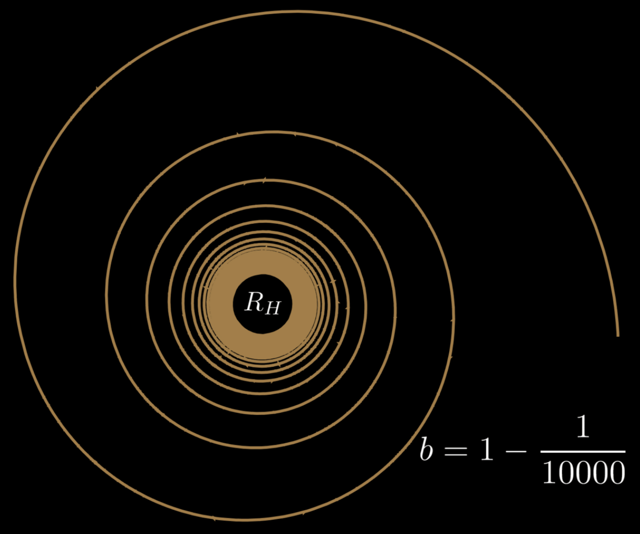
Aretakis helps preserve temporal conformal symmetry on the boundary in an interacting theory







Outlook

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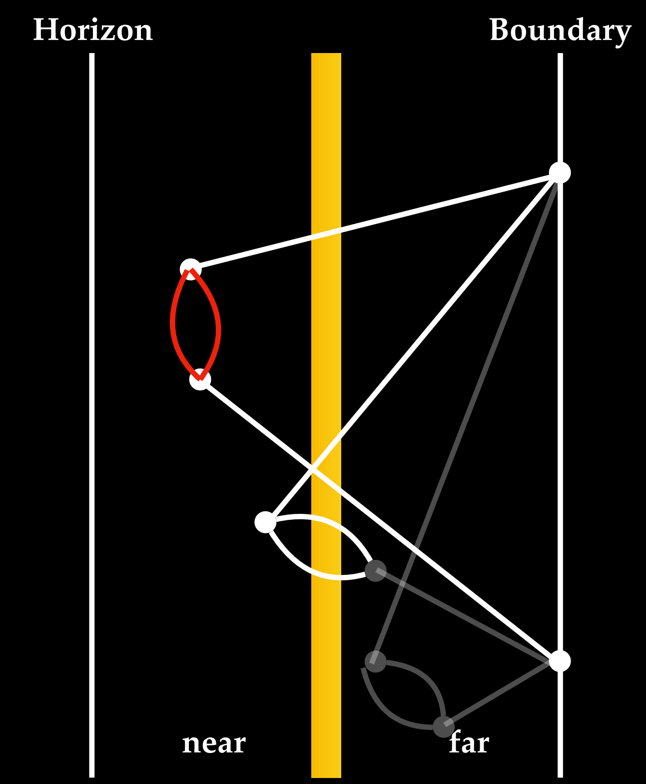
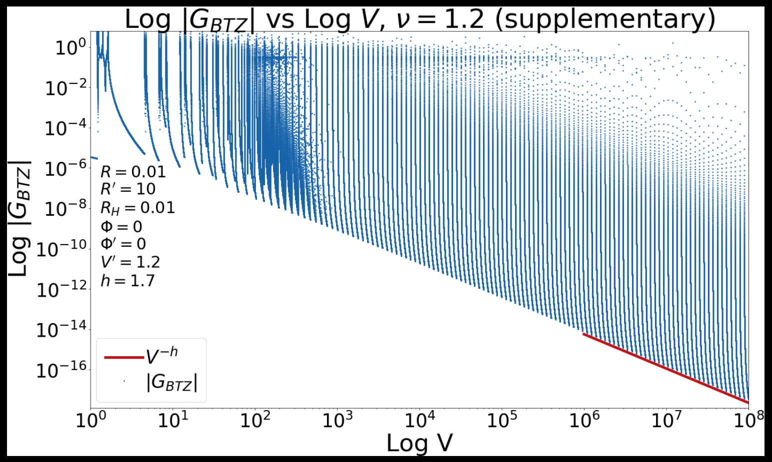
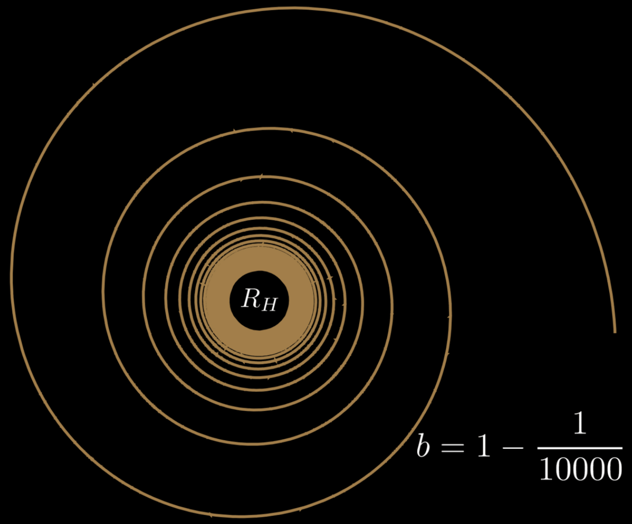
Holographic meaning of the Aretakis instability remains a mystery







Outlook

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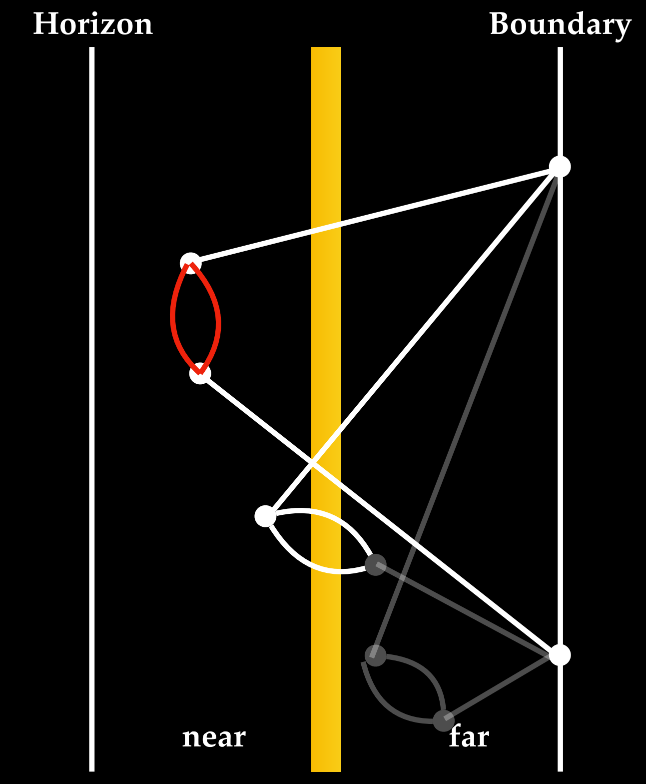

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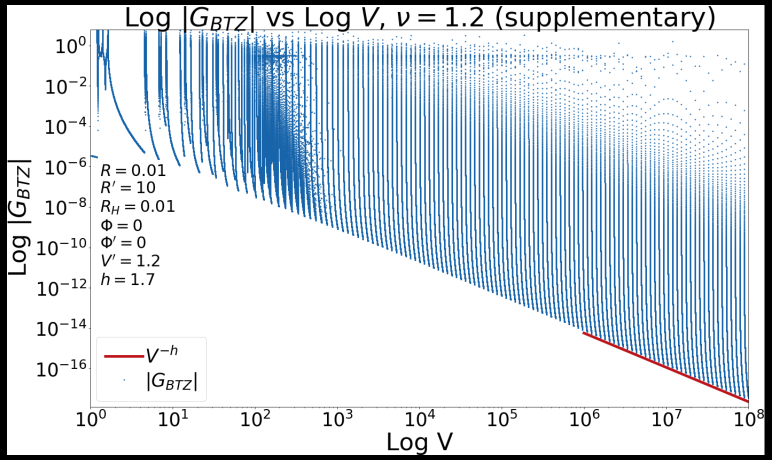

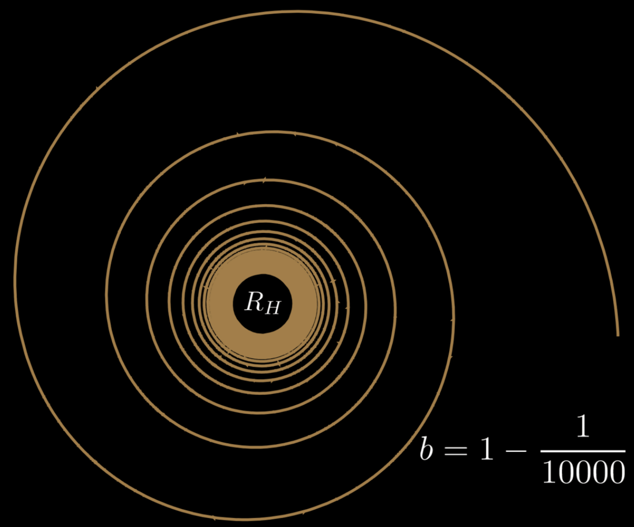

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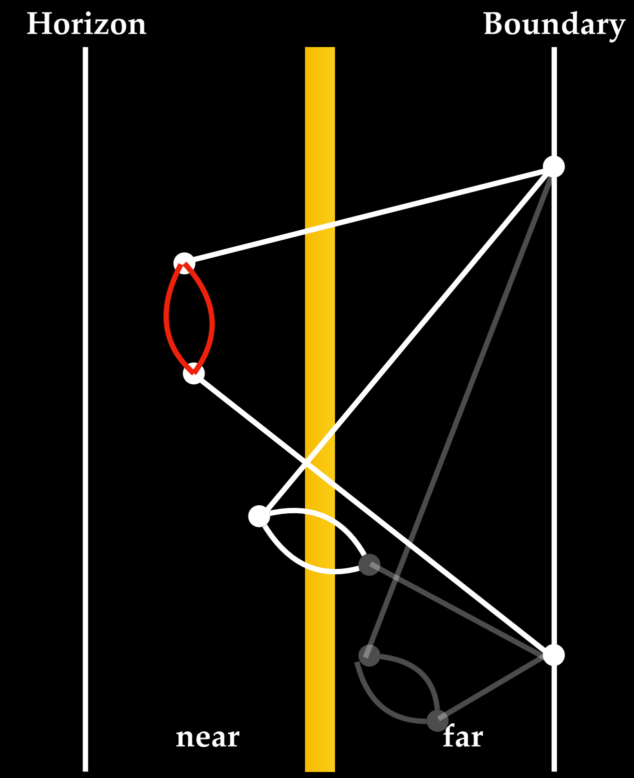

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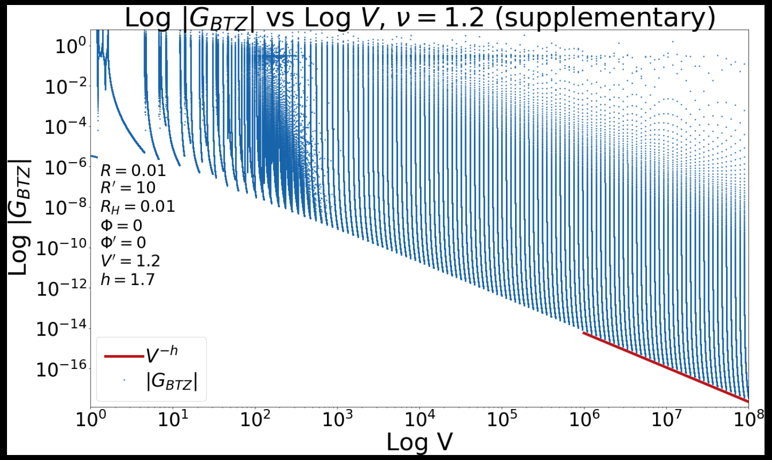

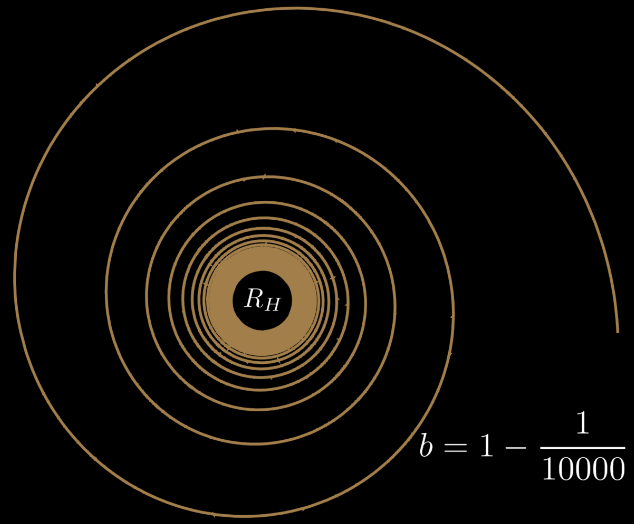

Null geodesics in the near horizon region seem to be important