Critical Behavior of Extremal Black Hole Perturbations. III. Holography?

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Caltech

with

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Aretakis instability in extreme black holes

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Transverse derivatives of perturbations grow ***at late times on the horizon***

Scalar perturbation

$$\boxed{\partial_x^n \phi \sim V^{-P+n} f(Vx)}$$

Ingoing time

Radial coordinate

as $V \to \infty$ and $x \to 0$

Late times

Horizon

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Holographic signature?

No explicit example studied in asymptotically *AdS*

2+1 dimensional rotating black hole - BTZ

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Simplest asymptotically AdS black hole to study - BTZ black hole

(Bañados-Teitelboim-Zanelli)

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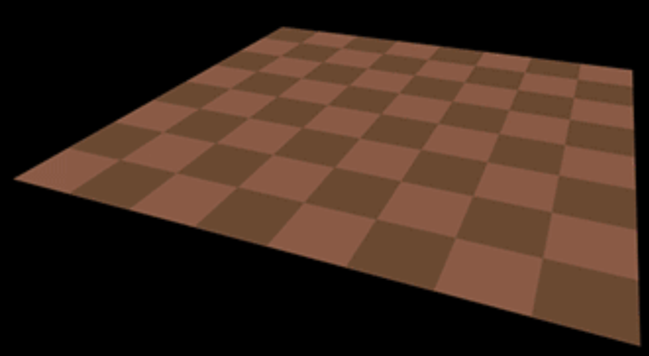

BTZ metric

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$$ds^2 = -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{\ell^2 r^2} dt^2 + \frac{\ell^2 r^2}{(r^2 - r_+^2)(r^2 - r_-^2)} dr^2 + r^2 \left(d\phi - \frac{r_+ r_-}{\ell r^2} dt \right)^2$$

Inner horizon

Outer horizon

AdS length scale

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AdS length scale

$$\phi \sim \phi + 2\pi$$

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This makes it a black hole and not just AdS_3

Green’s function for a scalar perturbation in extreme BTZ

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In ingoing, co-rotating coordinates $$G(x^\alpha, x^{\alpha'}) = \sum_m e^{im\Delta\varphi} G_m(\Delta V, x, x')$$

where $\Delta V = V - V'$ and $\Delta\varphi = \varphi - \varphi'$

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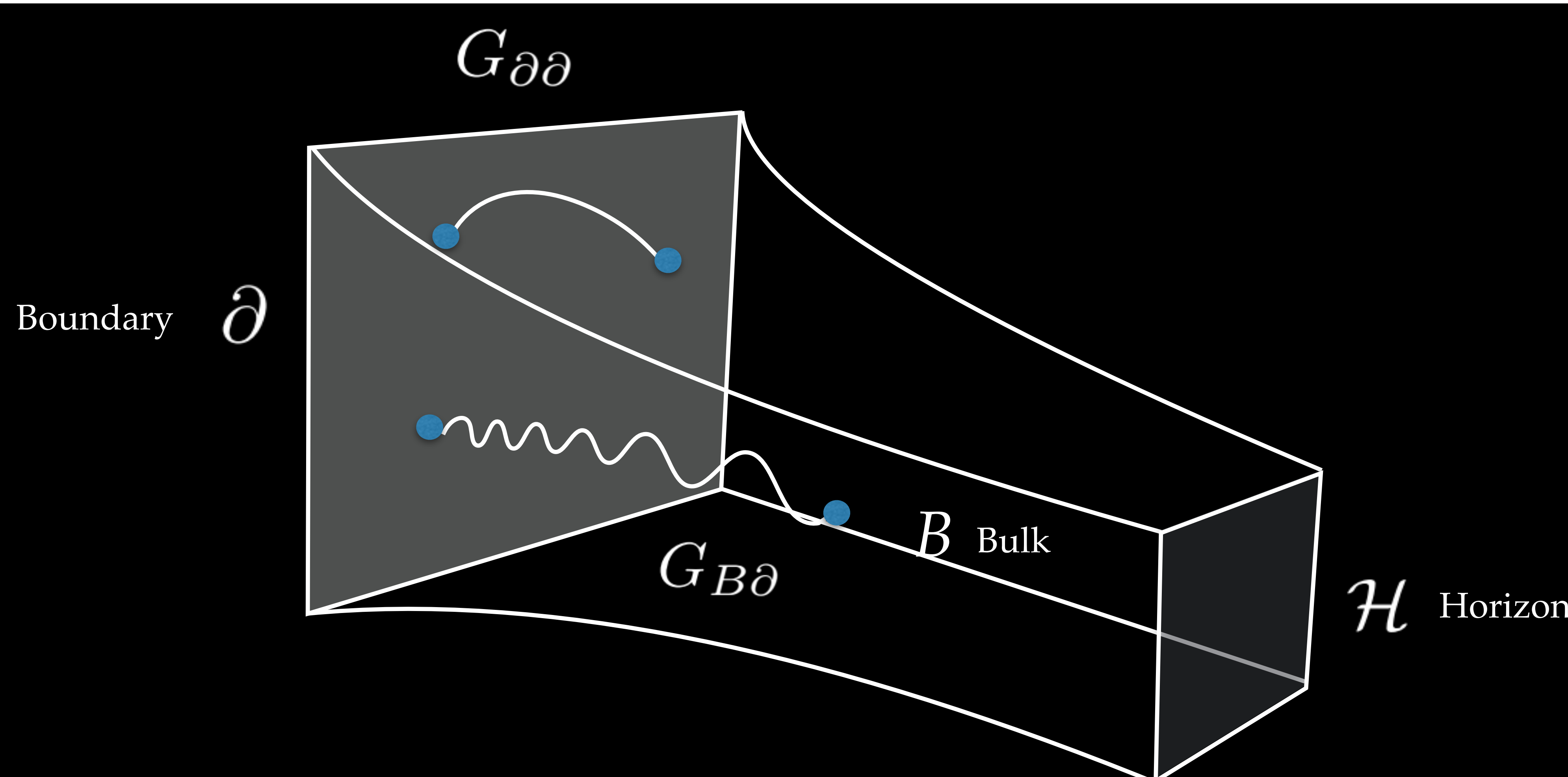
Boundary conditions - Ingoing waves at horizon & Dirichlet at boundary

Boundary propagators



$$G_{\partial\partial} \equiv \lim_{x\to\infty} \lim_{x'\to\infty} (xx')^h G$$

Boundary-Boundary propagator

$$G_{B\partial} \equiv \lim_{x'\to\infty} x'^h G$$

Bulk-Boundary propagator

where

$$h = \frac{1}{2}\left(1 + \sqrt{1 + \ell^2\mu^2}\right)$$

Results

Boundary-Boundary propagator for late times

$$G_{\partial\partial}(\Delta V, \Delta\varphi) = \frac{-1}{4\pi r_0} \frac{\Delta V^{-2h}}{\Gamma(2h)} \sum_m e^{i\Delta\varphi} \frac{\Gamma(h - im)}{\Gamma(1 - h - im)}$$

Bulk-Boundary propagator for late times

$$G_{B\partial}(x, \Delta V, \Delta\varphi) = \frac{-1}{4\pi r_0} \frac{\Delta V^{-h}(1 + \Delta V x)^{-h}}{\Gamma(2h)} \sum_m \frac{\Gamma(h - im)}{\Gamma(1 - h - im)}$$

$$\text{x} \ \exp\left(im \left(\frac{1 + \Delta V x}{(1 + x + \sqrt{1 + 2x})\Delta V} \right)\right)$$

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$$\boxed{V^{-2P}}$$

P is related to the mass of the scalar field

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$$\times \exp\left(im\left(\frac{1+\Delta V x}{(1+x+\sqrt{1+2x})\Delta V}\right)\right)$$

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Sum over *m* turns into an integral over *m*

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$$I = \frac{1}{2\pi}\int_{-\infty}^{\infty} dm \frac{\Gamma(h-im)}{\Gamma(1-h-im)}\exp\left(im\left(\frac{1+\Delta V x}{(1+x+\sqrt{1+2x})\Delta V}\right)\right)$$

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$$m = -i(h+n)$$

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$$\boxed{\begin{array}{c}\text{Poles at} \\ m = -i(h+n)\end{array}}$$

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$$I = e^{h\Delta\varphi} \left(\frac{1 + \Delta V x}{(1 + x + \sqrt{1 + 2x})\Delta V} \right)^h {}_0\tilde{F}_1 \left(-1 - 2h; -\left(\frac{1 + \Delta V x}{(1 + x + \sqrt{1 + 2x})\Delta V} \right) e^{\Delta\varphi} \right)$$

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Exactly cancels out what causes the Aretakis effect!!

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<u>'Planar BTZ' - Bulk-Boundary propagator for late times</u>

$$G^{p}_{B\partial}(x,\Delta V,\Delta\varphi) = \frac{-\Delta V^{-2h}}{2r_0}(1+x+\sqrt{1+2x})^{-h}$$

$$\times\; e^{h\Delta\varphi}\,{}_0\tilde{F}_1\left(-1-2h; -\left(\frac{1+\Delta V x}{(1+x+\sqrt{1+2x})\Delta V}\right)e^{\Delta\varphi}\right)$$

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- What is the CFT dual to the Aretakis instability? (4-point function?)
- Does the Aretakis instability occur in more general planar black holes?