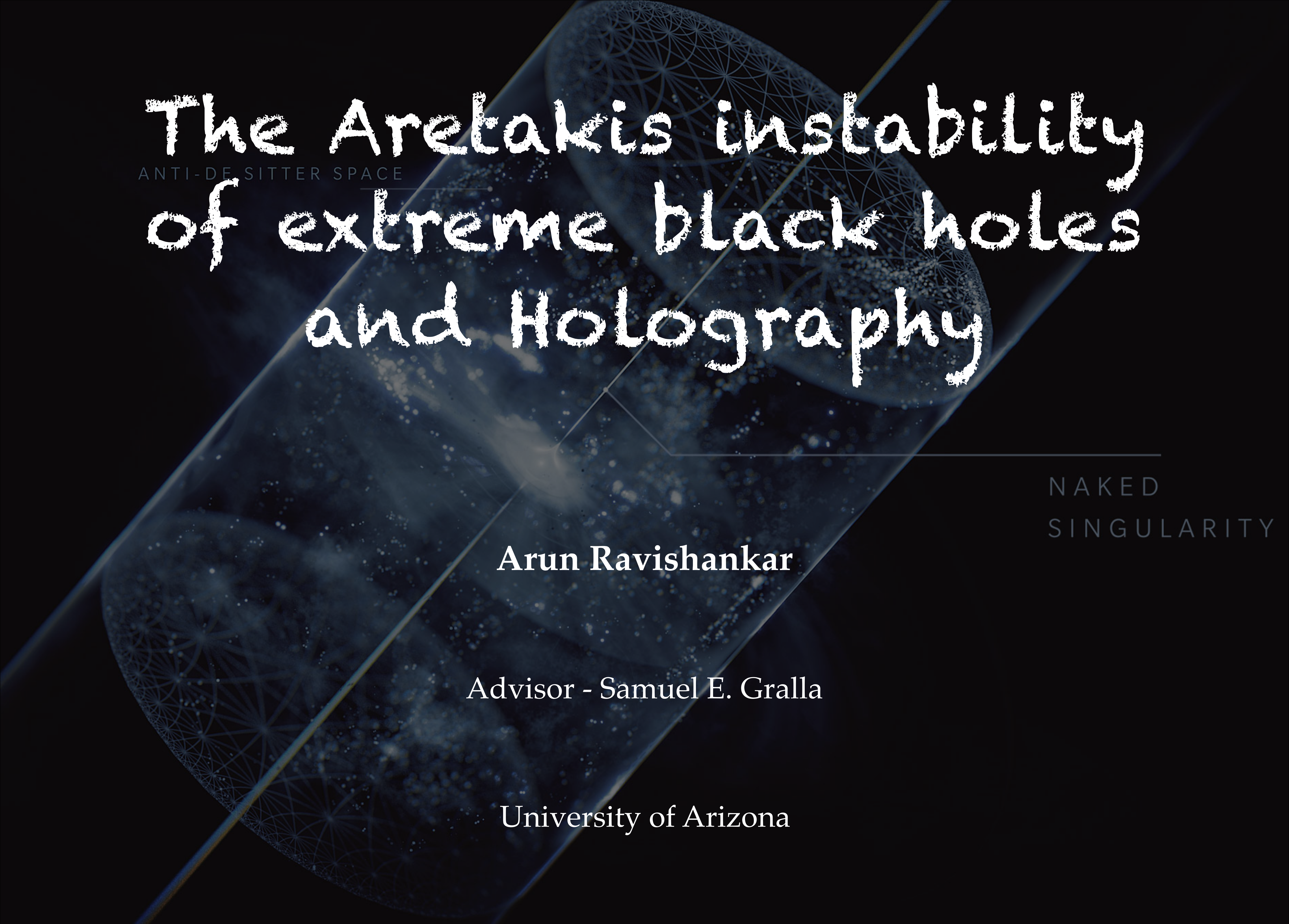

The Aretakis instability of extreme black holes and Holography

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Outline

Interesting phenomenon in the study of Black hole perturbations

Aretakis instability

- Instability of extreme black holes

Max spin/ charge for a given M of BH

- Theorem confirms instability for spacetimes with a compact horizon
- No explicit example with a non-compact horizon studied!
- Planar horizon topology for black holes in Anti-de-Sitter space relevant for holography

Constant negative curvature

Planar Reissner-Nordström Anti-de-Sitter (pRNAdS) black hole in 5 D

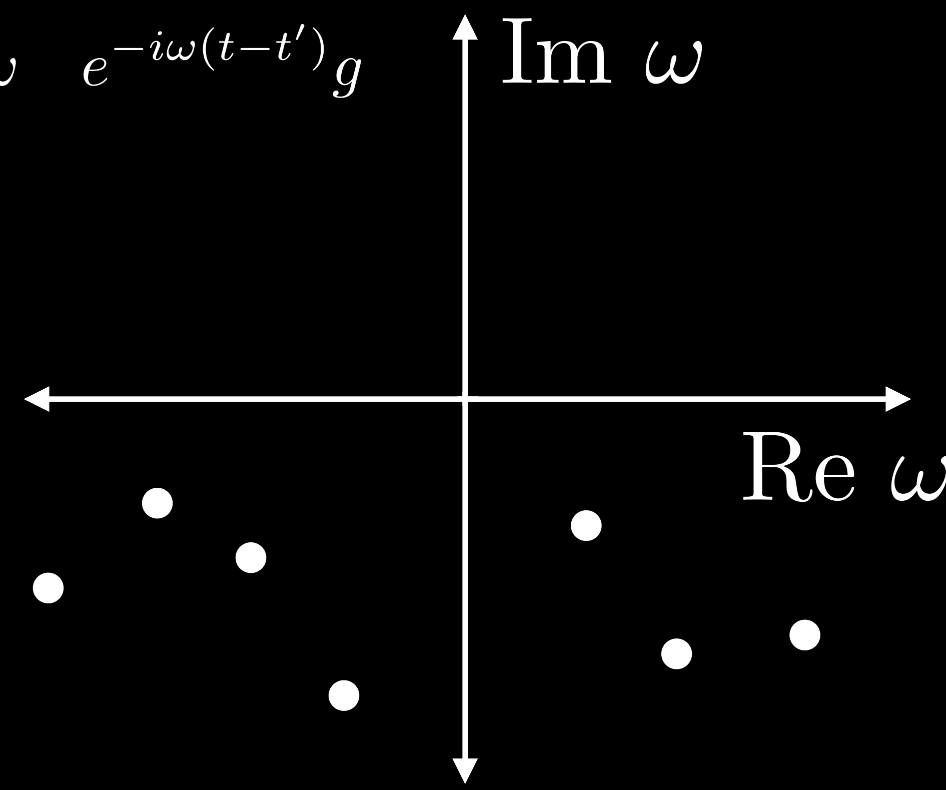
Bañados-Teitelboim-Zanelli black hole in 3D

- What consequences does the Aretakis instability have for holography?

Black Hole stability

Perturbations on black hole spacetime background

Mode stability $(\Box - \mu^2)G = \delta$ $G = \int d\omega \quad e^{-i\omega(t-t')} g$



Linear stability stronger result

Historically, Mode stability => Linear stability

Aretakis!

Aretakis

Axisymmetric / uncoupled massless scalar field perturbations to extreme Kerr / Reissner-Nordström (Aretakis 2012)

spinning BH — charged BH

First order transverse derivatives of field on horizon don't decay

along radial direction

Higher order derivatives grow

Conservation law on the horizon

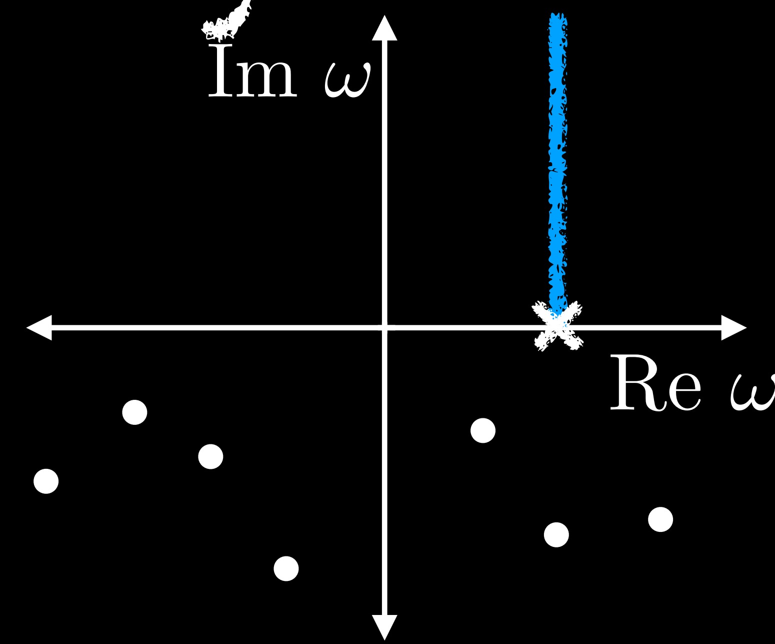
Depends only on ***local geometry of horizon*** and not global geometry

What followed

Other fields?	Scalar, electromagnetic, gravitational perturbations (Lucietti, Murata, Reall, Tanahashi 2013)
Non-axisymmetric massless scalar on extreme Kerr	Enhanced growth rate (Casals, Gralla, Zimmerman 2016)
Charged massless scalar on extreme RN	Same as above result (Zimmerman 2016)
Only extremal?	Transient growth at intermediate times (Gralla, Zimmerman, Zimmerman 2016)
Non-linear end state?	Generically non-extreme, fine-tuned initial perturbations - extreme BH (instability persists) (Murata, Reall, Tanahashi 2013)
General extreme horizons	Compact horizon topology (Lucietti, Reall 2012)

Understanding today

Branch point on complex ω plane



(Gralla, Zimmerman 2018)

Radial coordinate — Ingoing time

$$\psi \sim v^{-h} f_0(vx) \qquad x \to 0\,, \quad v \to \infty$$

Horizon — Late times

$$\text{Re } h \geq 1/2$$

$$h \equiv h(\mu, q)$$

$$\partial_x^n \psi \sim v^{-h+n} f_1(vx) \qquad x \to 0\,, \quad v \to \infty$$

On Horizon decay rate

$$v^{-h}$$

Off Horizon decay rate

$$v^{-2h}$$

Consequences?

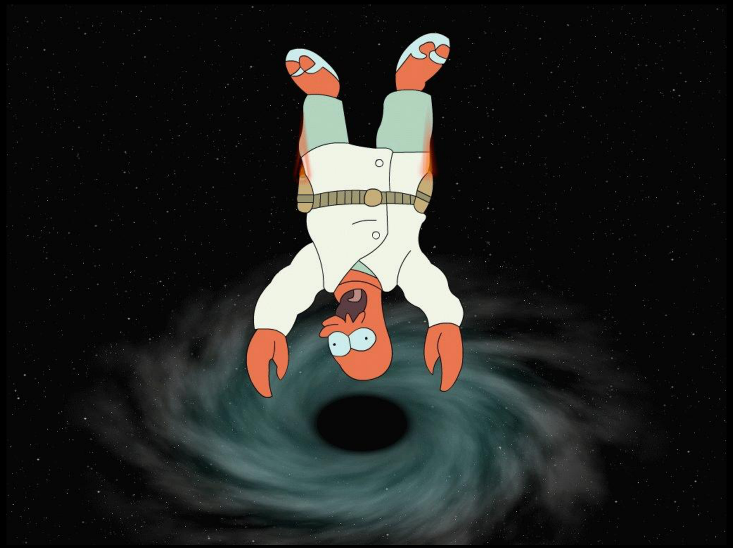
Aretakis - only on the future horizon

Not typical instability - scalar field itself decays

Only transverse derivatives of scalar field on the horizon grow at late times

Only Ingoing observers on the horizon see this effect

General covariance prevents observer independent quantities from becoming large



Field strength invariant	Kretschmann invariant	Squared electric field strength observed by infalling observer
$F^{\alpha\beta}F_{\alpha\beta}$	$R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$	$E^2 = F_{\mu\alpha}u^\alpha F^{\mu\beta}u_\beta$ (Gralla, Zimmerman 2018)
decays	decays	grows

Going forward

Universal framework for Aretakis exists

Theorem confirms instability for spacetimes with a compact horizon

No proof for non-compact horizon topologies

No explicit example with a non-compact horizon studied

Never studied in asymptotically Anti-de-Sitter spacetimes

A field of research that is interested in asymptotically AdS spacetimes - Holography

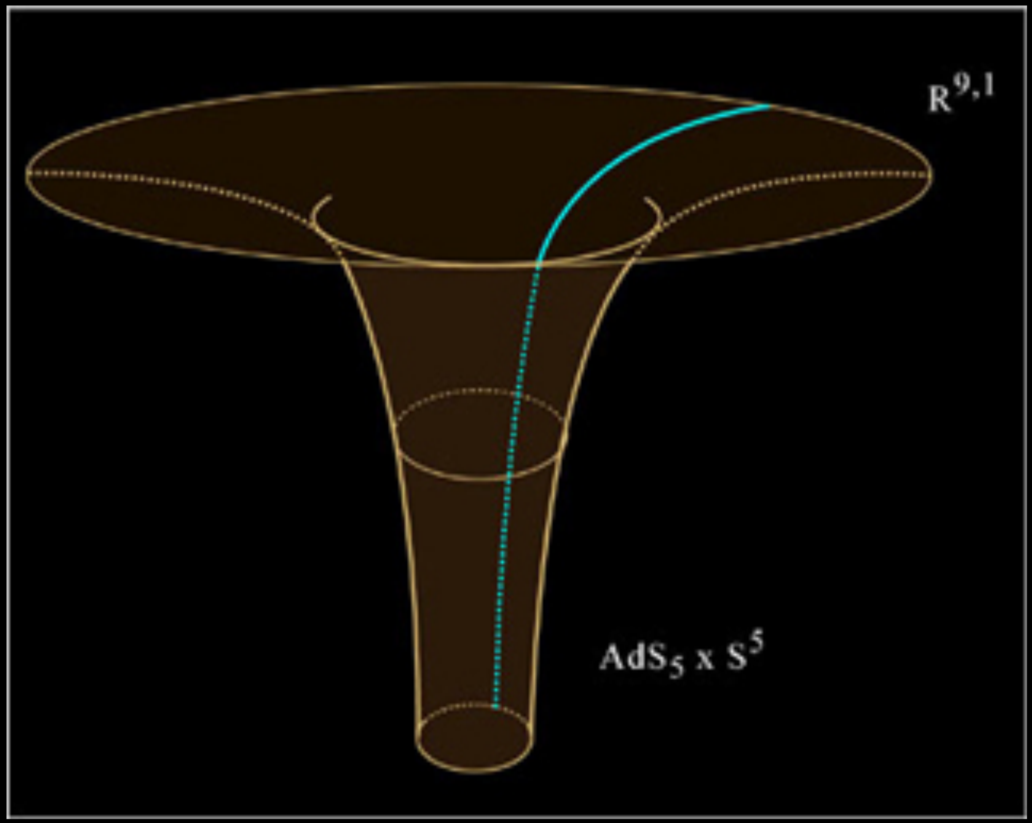
What consequences does the Aretakis instability have for holography?

Holography (Gauge-Gravity duality)

4*d* strongly coupled $\mathcal{N} = 4$ Super Yang Mills

$$=$$

IIB strings on $\mathrm{AdS}_5 \times S^5$



Theories of quantum gravity in *d + 1* dimensions $\leftrightarrow$ Non-gravitational conformal QFTs on the *d*-dimensional boundary

Field on boundary of gravitational system is related to operator of a CFT

Enough for us to work with to see what consequences Aretakis might have for holography

pRNAdS_5

Planar Reissner-Nordström
Anti-de-Sitter in $5d$

(Gralla, Ravishankar, Zimmerman 2018)

Maxwell field coupled to AdS gravity in 5 dimensions

$$ds^2 = \frac{\ell^2}{z^2}\left(-f d\tau^2 + \frac{dz^2}{f} + d\vec{y}^2\right)$$

Parameters of black hole - charge density and AdS length

$$f = 1 - 3z^4(1-2\sigma) + 2z^6(1-3\sigma)$$

Re-parameterize to make temperature a parameter

$$T \propto \sigma \to 0$$

Extremal limit

Boundary	Horizon
$z \to 0$	$z \to 1$

$$1 - z \sim x \qquad x \ll 1$$

Framework of Calculations

$$(D^2 - \mu^2)\psi = 0$$

details of geometry

$$D^\mu = \nabla^\mu - iqA^\mu$$

coupled to gauge field

$$(D^2 - \mu^2)G = \delta$$

Ansatz

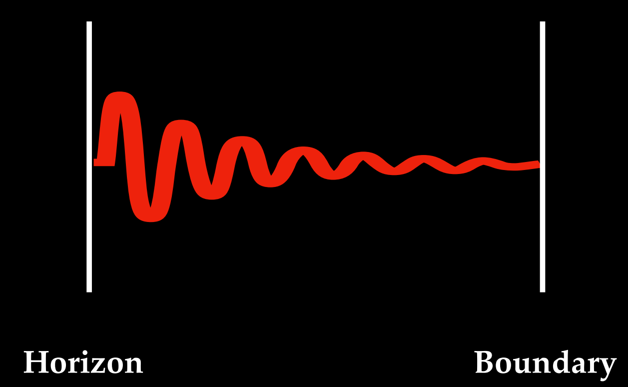
$$G = \int \frac{d\omega d^3k}{(2\pi)^4} e^{-i\omega t + i\vec{k}.\vec{y}} g(\vec{k}, \omega, x)$$

$$\hat{O}g = \delta$$

ODE for g

Ingoing waves on horizon

$$R_{\text{in}} \sim e^{-i\omega r_*} \qquad x \to 0$$



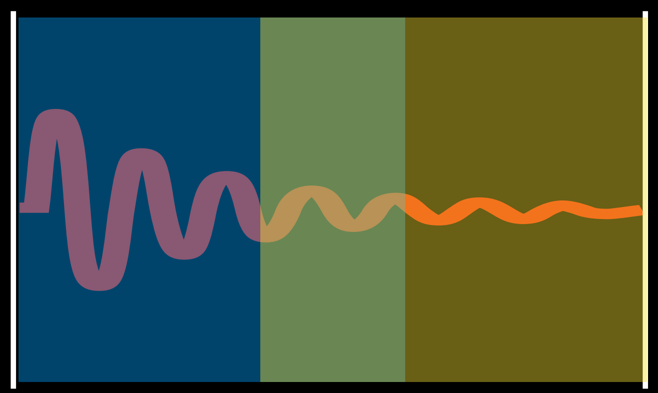
Appropriate decay at boundary of AdS

$$R_{\text{far}} \sim x^{-\Delta_+} \qquad x \to \infty$$

Framework of Calculations

$\hat{O} g = \delta$ Usually not tractable analytically

Matched asymptotic expansions



$$G_k \equiv L^{-1}[g] \sim v^{-h} f(vx, \vec{k}) \qquad v \to \infty \ , \ x \to 0$$

Aretakis pops out!

$\omega \ll 1$ Late times

Near region $1 - z \ll 1$

Overlap region $\omega \ll 1 - z \ll 1$

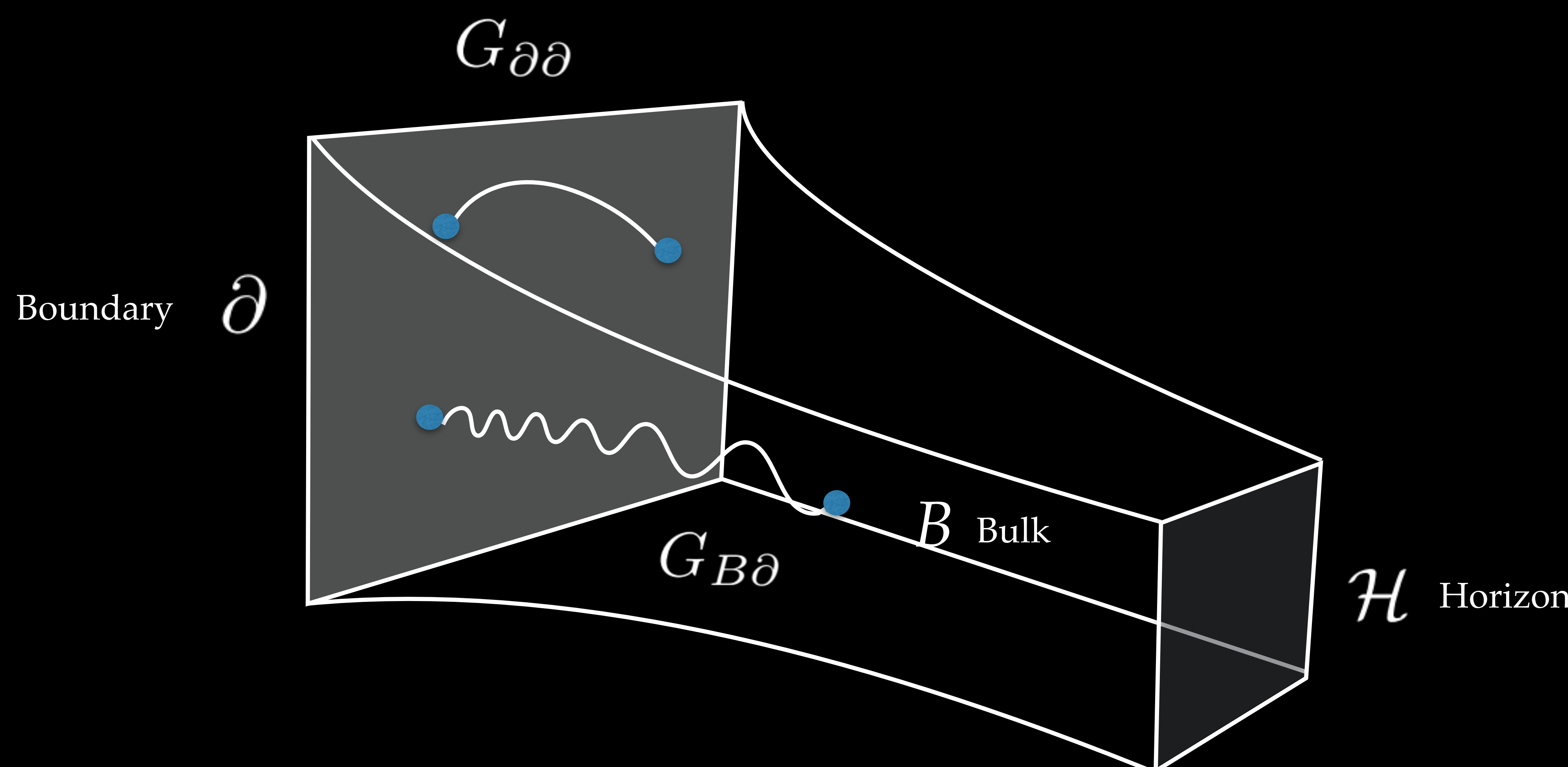
Far region $1 - z \gg \omega$

$$g = \frac{R_{\rm in}(x_<) R_{\rm far}(x_>)}{\mathcal{W}}$$

Wronskian

$$G = \int d^3k \quad G_k$$ if tractable

Boundary propagators



$$G_{\partial B}(\tau, y, z) = \ell^{3/2} \lim_{z' \to 0} (z')^{-\Delta_+} G$$

Bulk-Boundary propagator

$$G_{\partial\partial}(\tau, y) = \ell^3 \lim_{z \to 0} \lim_{z' \to 0} (zz')^{-\Delta_+} G$$

Boundary-Boundary propagator

where

$$\Delta_\pm = 2 \pm \sqrt{4 + \ell^2 \mu^2}$$

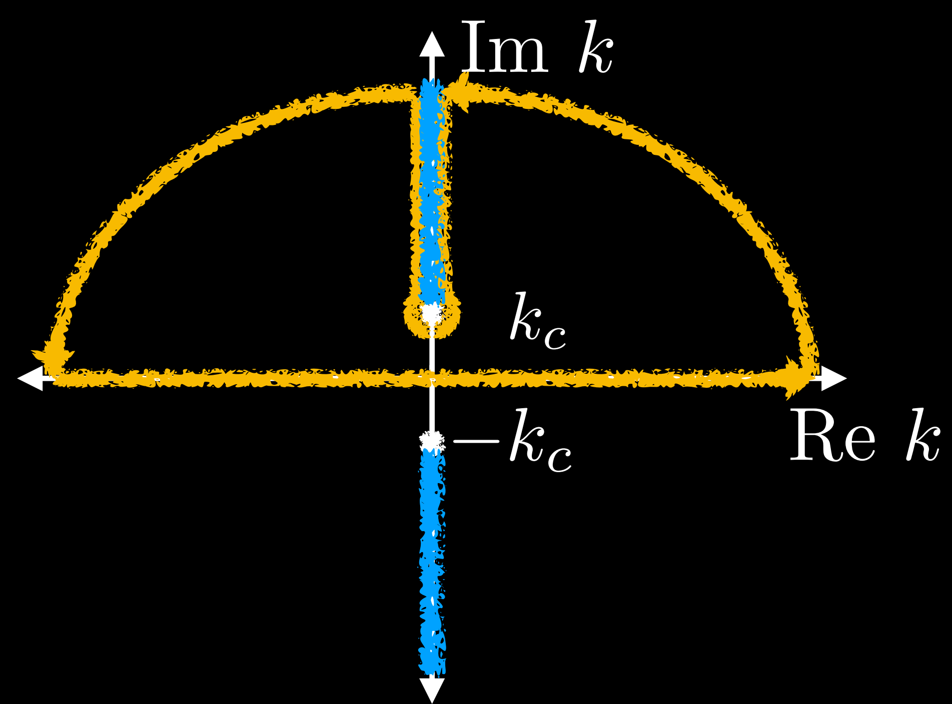
Green's function

$$G_{\partial B} = \frac{1}{\ell^{3/2}} \int \frac{d^3k}{(2\pi)^3} \frac{-e^{i\vec{k}.\vec{y}}\Gamma(h - i\hat{e})}{2\Gamma(2h)\Gamma(1-h-i\hat{e})B_+} (6v)^{-h-i\hat{e}}(1+6(1-z)v)^{-h+i\hat{e}}$$

$$h = \frac{1}{2} + \nu \qquad \nu = \nu(k, \ell, \mu, q) \qquad B_+ = B_+(k, q, \ell, \mu)$$

Parameter range - BH is mode stable $\nu > 0$ This range - no poles in k

Branch points at $\pm k_c$



Results

$$G^{\text{near}}_{\partial B} \sim \frac{\mathcal{C}}{\ell^{3/2} y}\left(\frac{k_c}{y}\right)^{3/2} e^{-yk_c}(6v)^{-1/2-i\hat{e}}(1+6v(1-z))^{-1/2+i\hat{e}}, \qquad y \to \infty$$

$$G^{\text{far}}_{\partial B} \sim \frac{\mathcal{C}}{\ell^{3/2} y}\left(\frac{k_c}{y}\right)^{3/2} e^{-yk_c} t^{-1} R_{\text{far}}\,|_{h=1/2}\,, \qquad y \to \infty$$

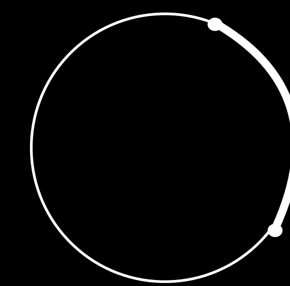
Aretakis persists after k-integral in a spacetime with non-compact horizon topology!

As expected, asymptotic AdS space doesn’t affect Aretakis

What is the CFT dual to Aretakis?

CFT dual?

Don't see anything in a 2 point function on the boundary



Aretakis is seen only on the Horizon of the black hole

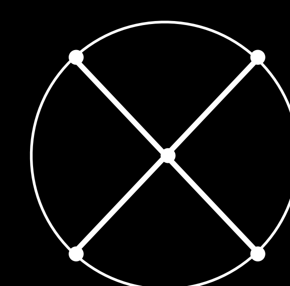
On Horizon decay rate

$$v^{-h}$$

Off Horizon decay rate

$$v^{-2h}$$

Interacting theory - integrate through the bulk to include near-horizon effects



Do we see a signature for Aretakis for a CFT that lives on the boundary?

Boundary correlators grow or decay slower than expected?

Scaling arguments

Temporal conformal transformations

late times $t \to \lambda^{-1} t, \quad \mathcal{O} \to \lambda^{1/2} \mathcal{O}$ $\quad \langle \mathcal{O}\mathcal{O} \rangle \sim G^{far}_{\partial\partial} \sim t^{-1}, \qquad t \to \infty$

At tree level,

$\langle \mathcal{O}_1 \mathcal{O}_2 \rangle \to \lambda \langle \mathcal{O}_1 \mathcal{O}_2 \rangle$ $\quad \langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle \to \lambda^2 \langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle \quad \text{under} \quad t_i \to \lambda^{-1} t_i$

late times

Given that the scaling of $G^{\text{near}}_{\partial B}$ is different, do we see a signature from near-horizon region?

$$G^{\text{near}}_{\partial B} \sim v^{-1/2}, \quad 1 - z \to 0 \ , \ v \to \infty$$

We argue that only the near region matters in leading order at late times on the boundary

Temporal conformal symmetry of boundary operator preserved due to Aretakis!

Generalization

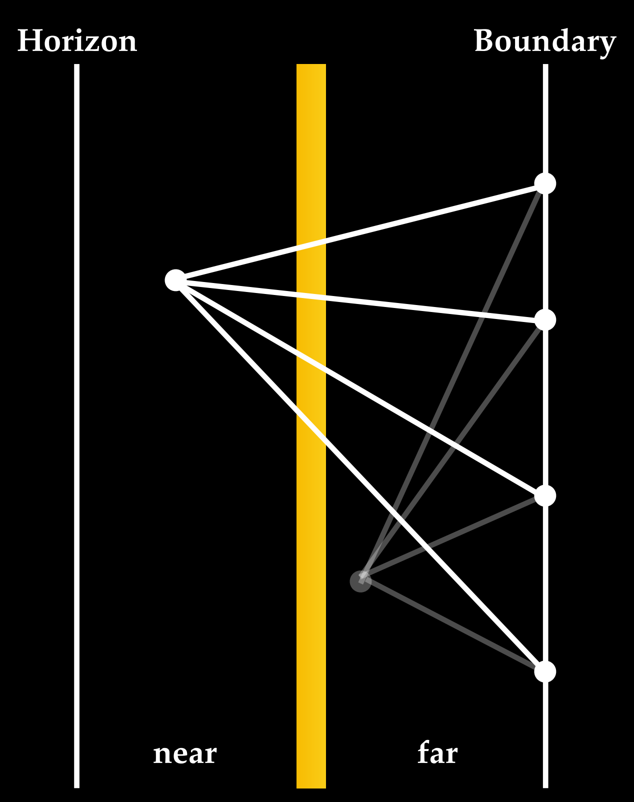
λ^0 for each near-region vertex

λ^{-1} for each far-region vertex

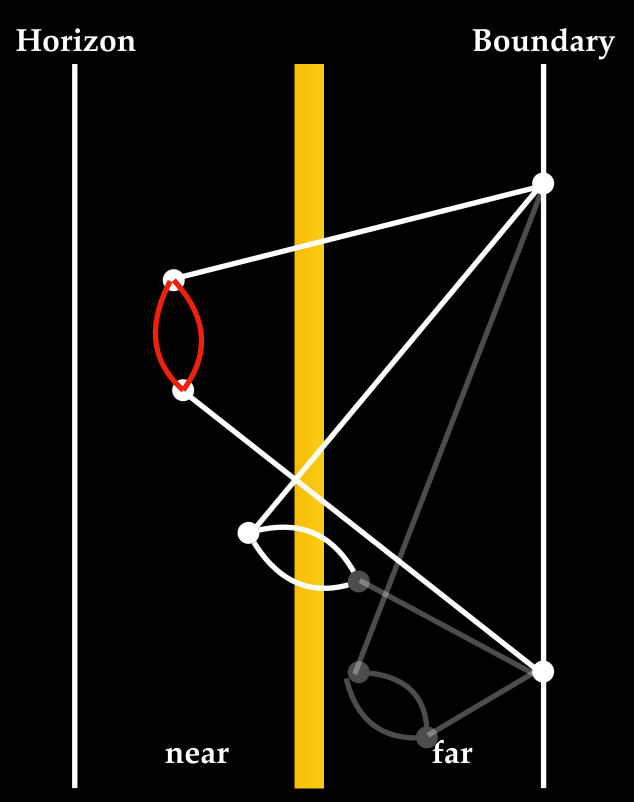
$\lambda^{1/2}$ for each boundary-near propagator

λ for each boundary-far propagator

λ^0 for each near-near propagator



$\Lambda\Phi^4$



$\Lambda\Phi^3$

Summary

Aretakis persists in spacetime with non-compact horizon topology

Aretakis persists in spacetime which is asymptotically AdS

Aretakis helps preserve temporal conformal symmetry on the boundary in an interacting theory

Holographic meaning of Aretakis instability remains a mystery

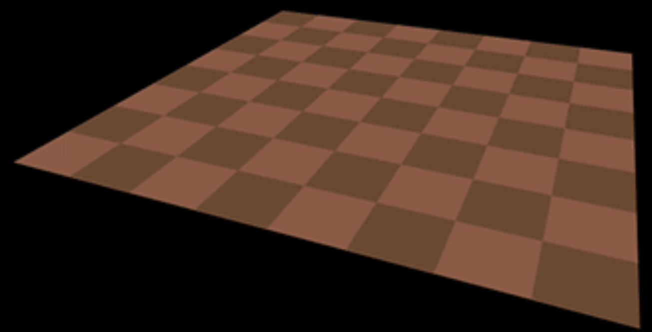
Bañados Teitelboim Zanelli (BTZ)

(ongoing)

Rotating black hole in $3d$ - asymptotically AdS

Gravity in $3d$ special - All vacuum spacetimes have constant curvature, locally

Periodically identify coordinates - different global geometries



Rotating black hole - similar properties to Kerr

We observe that the Aretakis instability persists

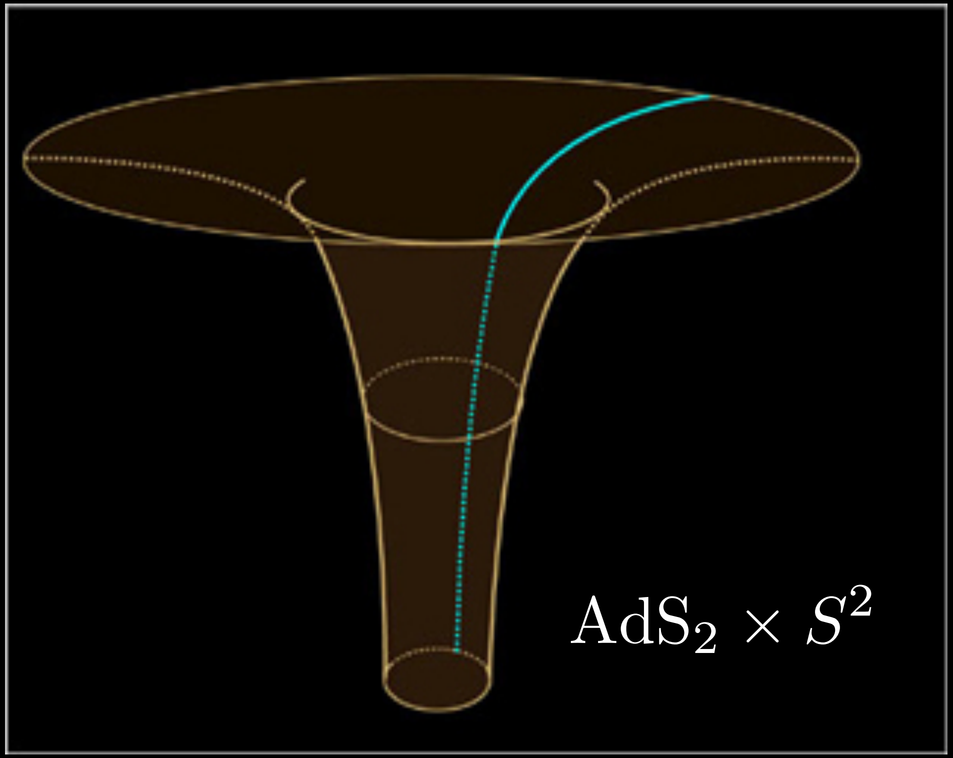
Interestingly, we preliminarily see that if initial data is not sufficiently smooth, the field is either not absolutely integrable, not of bounded variation or has infinite discontinuities in a finite interval

Going forward

Near horizon region of an extremal black hole - AdS_2 factor (Kunduri, Lucietti, Reall 2007)

Aretakis appears due to emergence of scaling symmetry in the near horizon region that appears due to the AdS_2 factor

(Gralla, Zimmerman 2018)



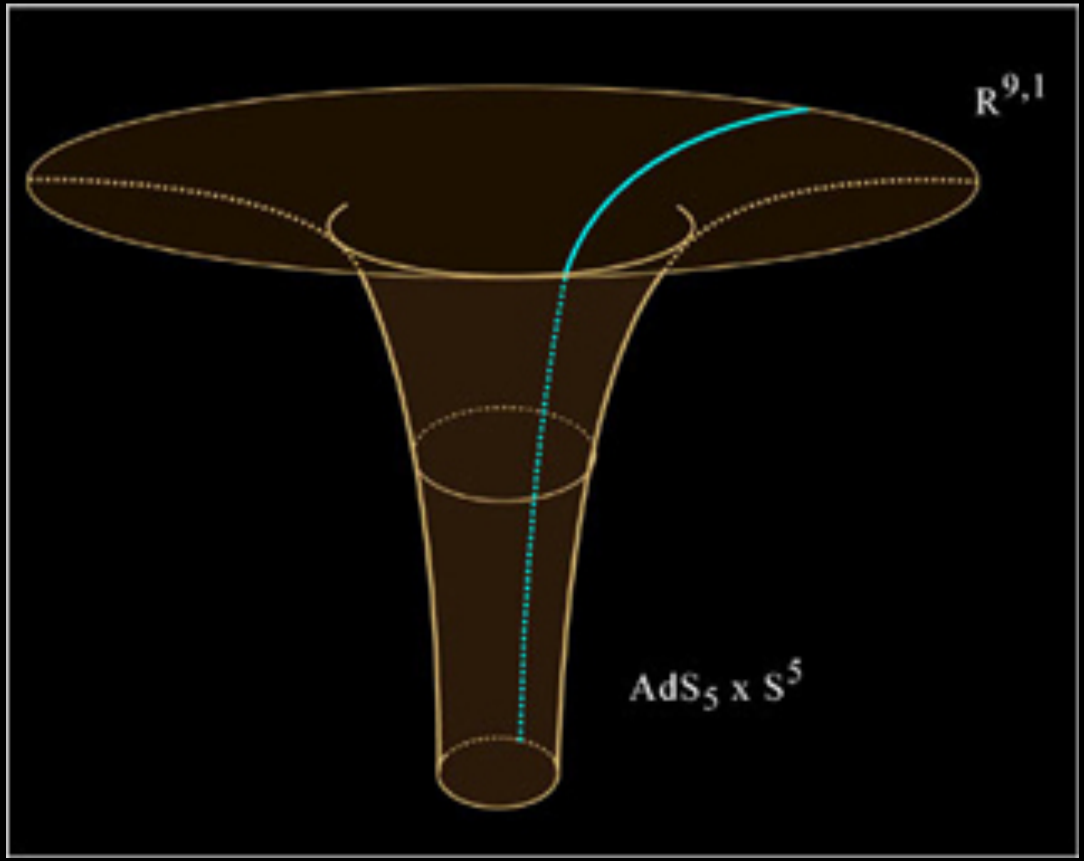
All extreme black holes studied have near horizon geometry of AdS_2

Universal framework for Aretakis now exists based on AdS_2

(Gralla, Zimmerman 2018)

What next?

D1-D3 brane - system in original statement of the correspondence



D1-D3 brane has a near horizon limit of $\mathrm{AdS}_5 \times S^5$

Does this system possess interesting late-time behavior? Does it have the Aretakis instability or something similar in a different limit?